无网格方法与近场动力学简介

损伤力学与结构非线性分析暑期研讨班

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The methods with simplicity (and fundamentally sound) win the game.

李杰 教授

朱伯龙 先生

理論聯系實際


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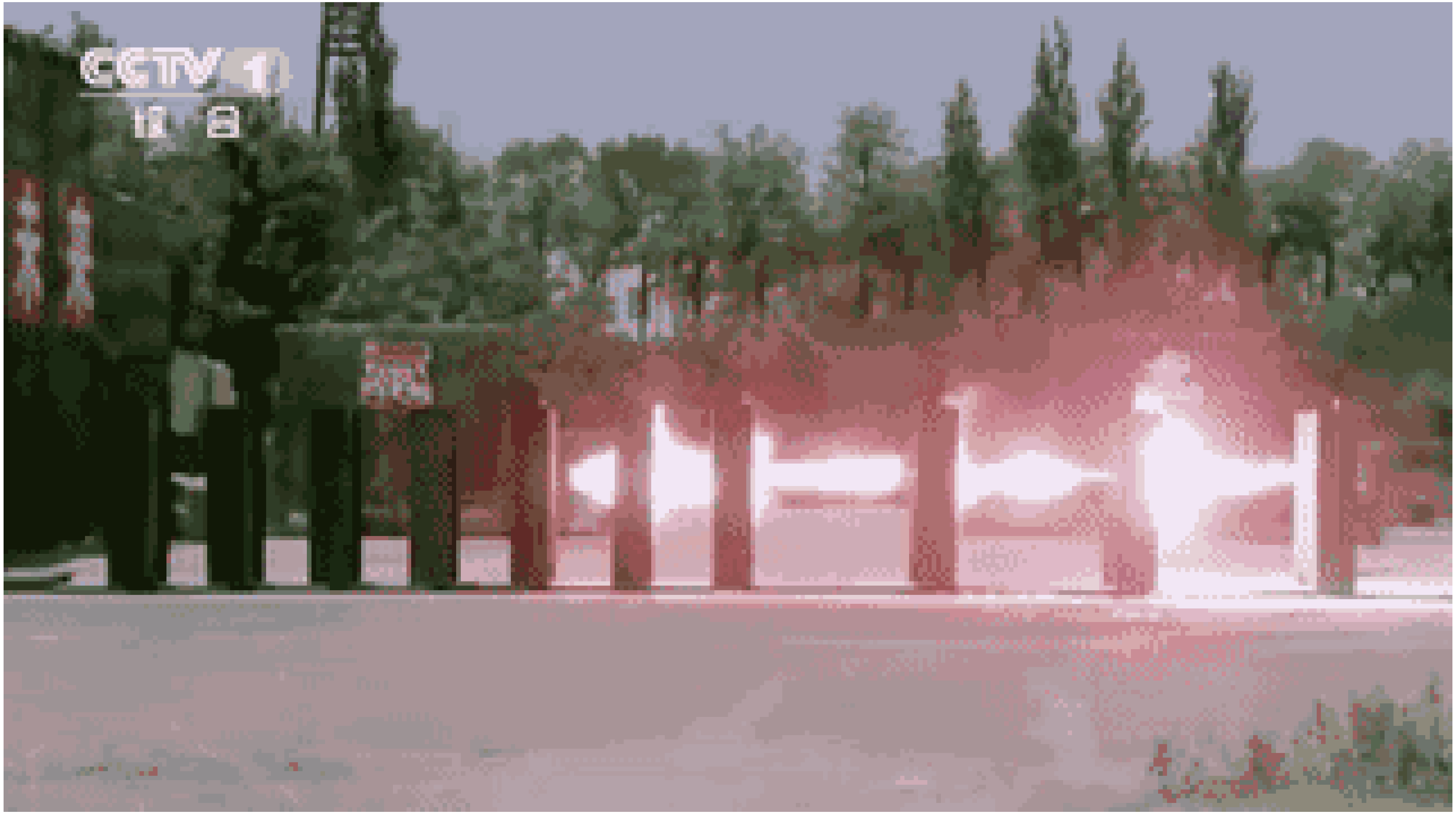

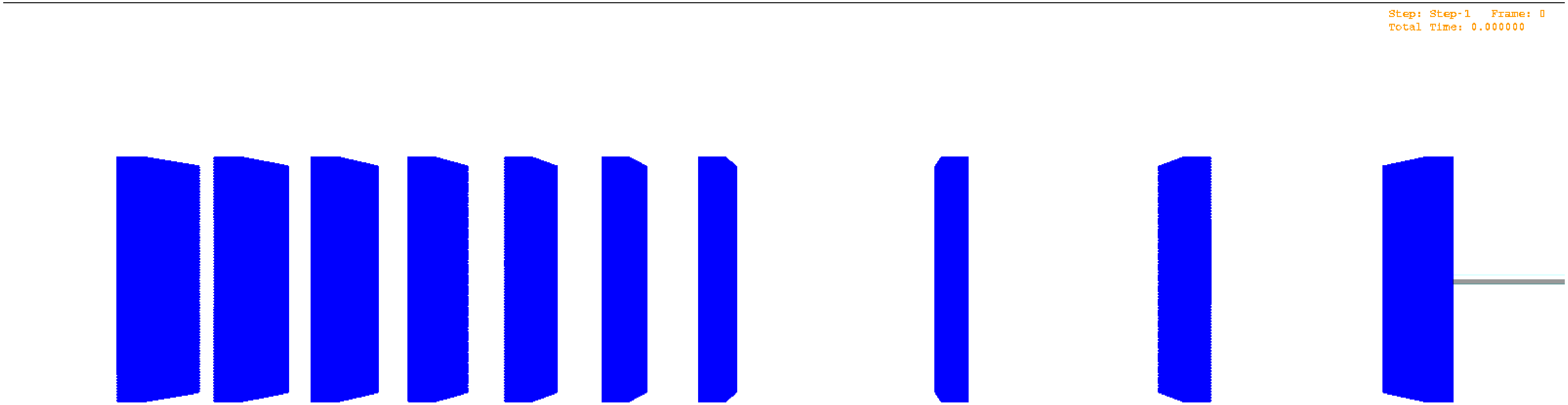
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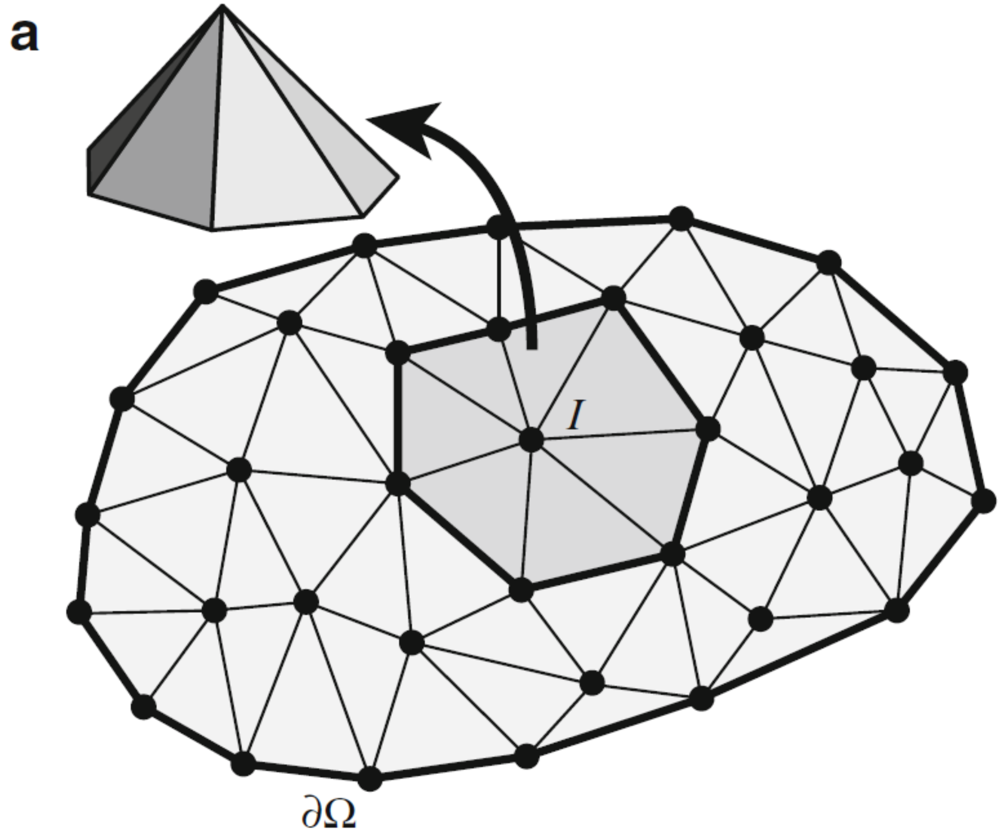
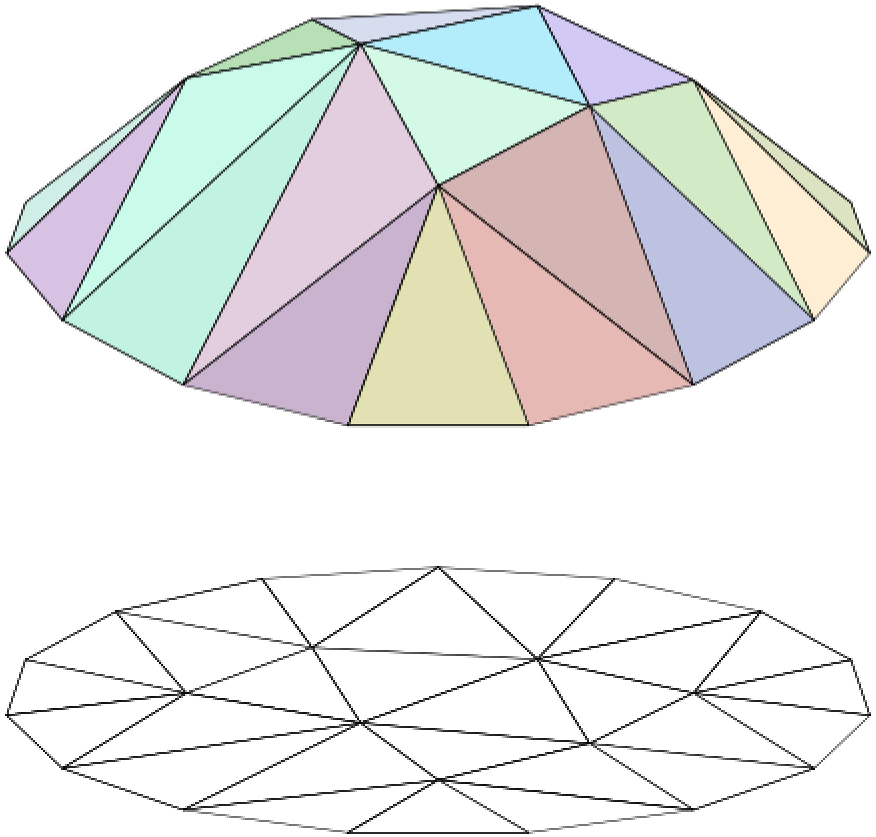
网格类计算方法

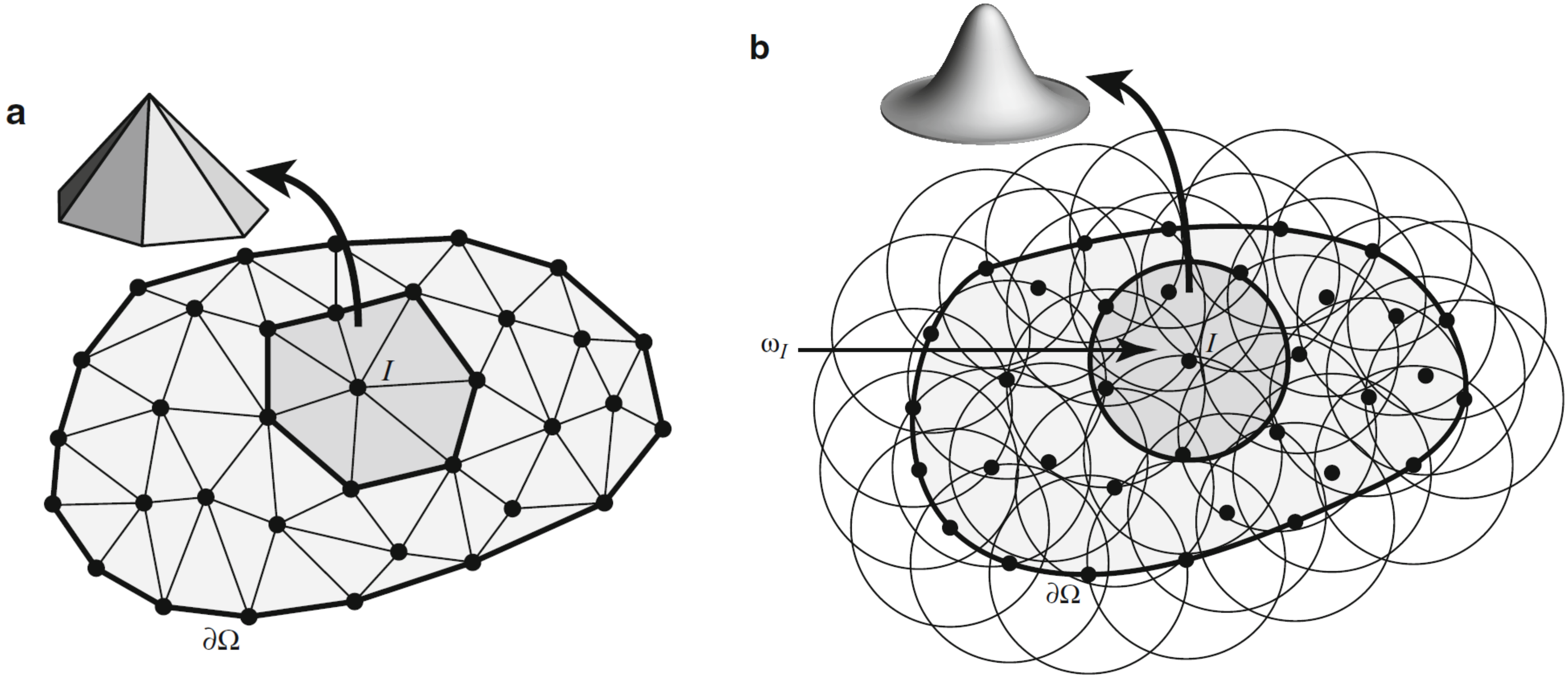




网格类方法的优缺点

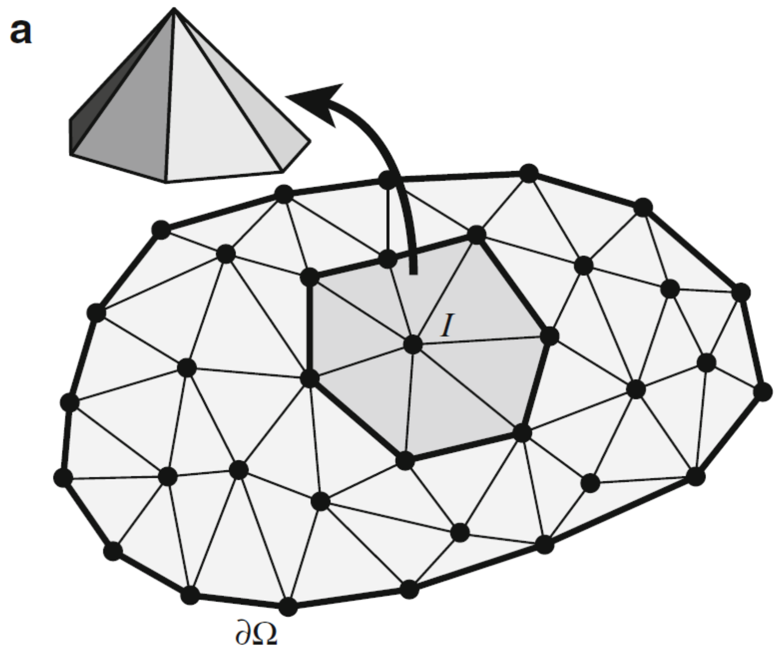
- 复杂几何体的网格构建很耗时
- 很难构造全场域高阶连续单元
- 很难进行高阶逼近操作
- 网格大变形与畸变问题
- 。。。。。。

考虑不用网格构造形函数



基于DELTA函数逼近构造函数空间

$$f(x) = \int_\Omega \delta(x-y) f(y) \, \mathrm{d}y \approx \sum_j \psi_j(x) f_j \Delta V_j$$

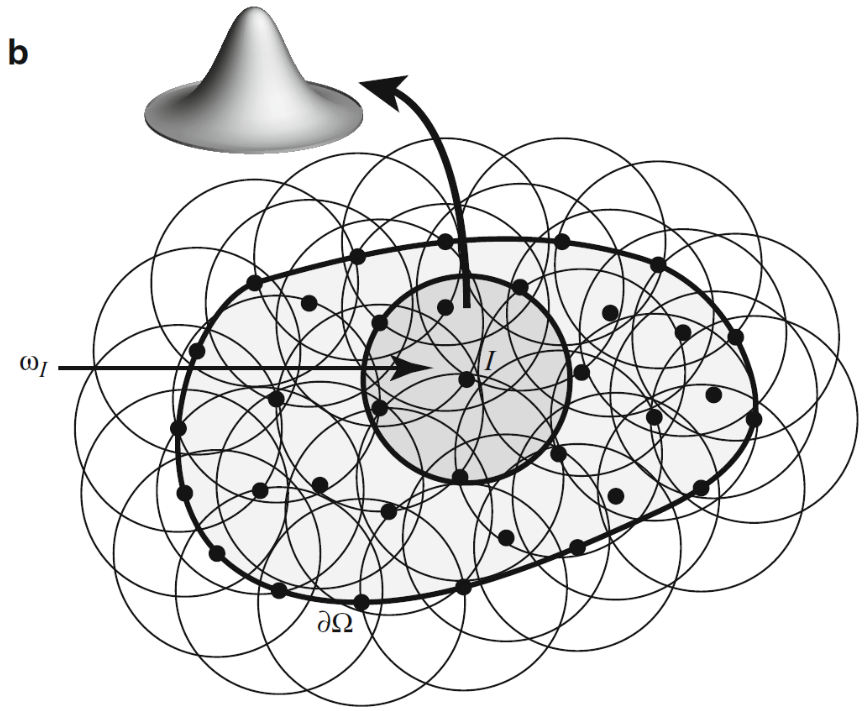


$$\psi_i(\mathbf{x}_j) = \delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

$$\Rightarrow \quad \hat{f}(\mathbf{x}_i) = \sum_j \psi_j(\mathbf{x}_i) f(\mathbf{x}_j) = f(\mathbf{x}_i)$$

基于DELTA函数逼近构造函数空间

$$f(x)=\int_\Omega \delta(x-y)\,f(y)\,\mathrm{d}y \approx \sum_j \psi_j(x)\,f_j\,\Delta V_j$$



插值函数也可不具备delta性质

$$\Rightarrow \quad \hat f(\mathbf{x}_i)=\sum_j \psi_j(\mathbf{x}_i)\,\tilde f(\mathbf{x}_j)= f(\mathbf{x}_i)$$

粒子类无网格方法！

Smoothed particle hydrodynamics (SPH) (1977)

Element-free Galerkin method (EFG / EFGM) (1994)

Reproducing kernel particle method (RKPM) (1995)

hp-clouds

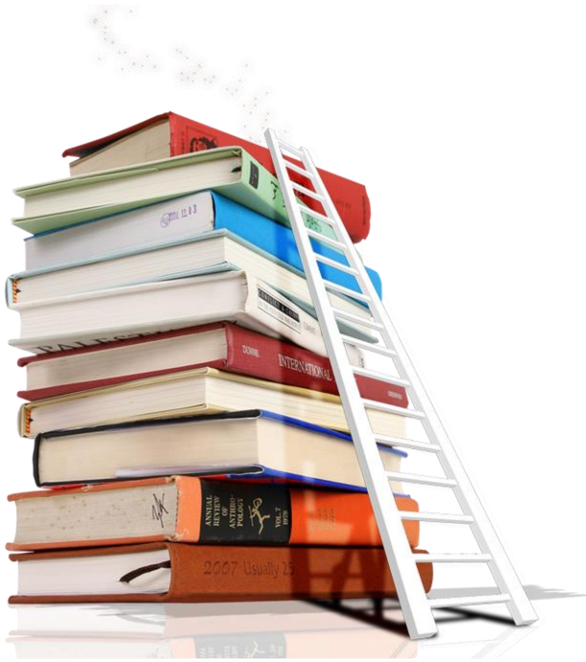
Material Point Method (MPM)

Meshless local Petrov Galerkin (MLPG)

Meshfree local radial point interpolation method (RPIM)

Radial Basis Integral Equation Method

and so on。。。。。。



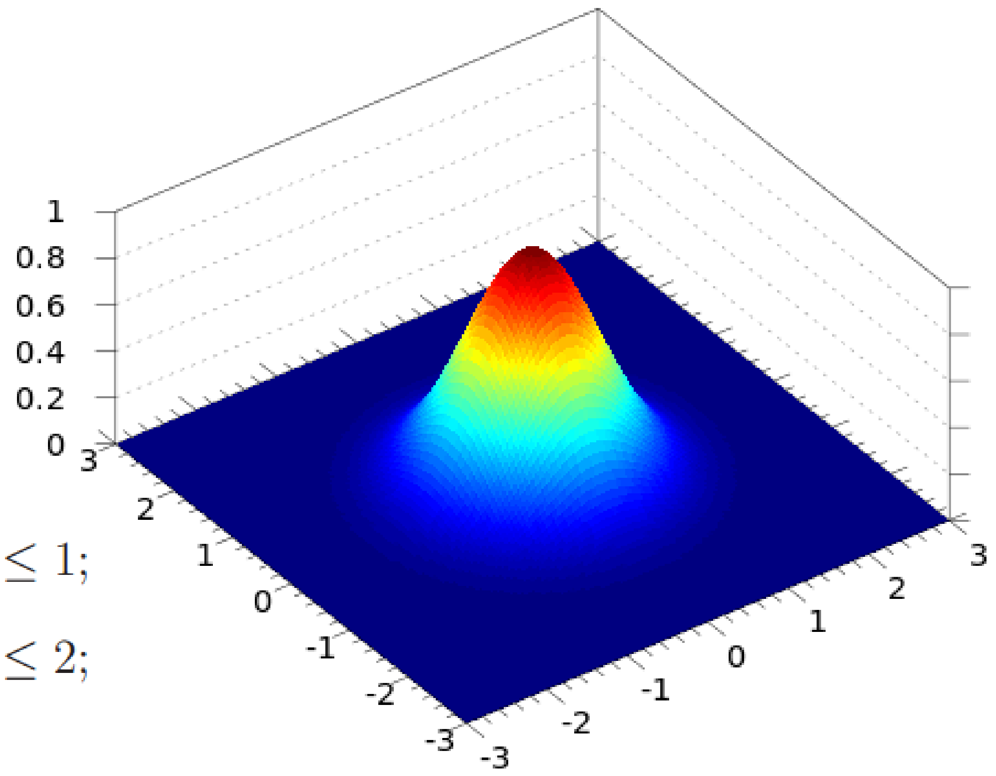
Smoothed Particle Hydrodynamics

- 最早是天体物理学家提出的，主要用于模拟星系的演化（Gingold and Monaghan 1977, Lucy 1977）.
- 后来研究者看到了这一类方法在连续介质模拟过程中的潜力，将其引入到流体和固体的数值模拟过程中（Hoover 2007）.
- 其中Libersky and Petschek(1990) 最早采用SPH模拟了材料强度破坏问题.

考虑基于核函数的逼近

$$\langle f(\mathbf{r})\rangle = \int W(\mathbf{r}-\mathbf{r}', h)\, f(\mathbf{r}')\,\mathrm{d}\mathbf{r}'$$

$$\delta(\mathbf{r}-\mathbf{r}')$$



$$W(r,h) = \frac{1}{\pi h^3}\begin{cases} 1-\frac{3}{2}x^2+\frac{3}{4}x^3 & 0\le x\le 1; \\ \frac{1}{4}(2-x)^3 & 1\le x\le 2; \\ 0 & x\ge 2, \end{cases}$$

离散形式的核函数的逼近

$$\langle f(\mathbf{r})\rangle = \int W(\mathbf{r}-\mathbf{r}', h)\, f(\mathbf{r}')\,\mathrm{d}\mathbf{r}'$$

$$\langle f(\mathbf{r})\rangle \approx \sum_j W(\mathbf{r}-\mathbf{r}_j, h)\, f(\mathbf{r}_j) V_j$$

$$= \sum_j W(\mathbf{r}-\mathbf{r}_j, h)\, f(\mathbf{r}_j)\frac{m_j}{\rho_j}$$

$$\rho(\mathbf{r}) = \sum_j W(\mathbf{r}-\mathbf{r}_j, h) m_j$$

林家翘（1916-2013）

位移场逼近

$$\mathbf{u}(\mathbf{r})=\sum_j W(\mathbf{r}-\mathbf{r}_j,h)\mathbf{u}_j\frac{m_j}{\rho_j}+\mathrm{O}(\mathbf{r}^2)$$

$$\approx\sum_j W(\mathbf{r}-\mathbf{r}_j,h)\mathbf{u}_j\frac{m_j}{\rho_j}$$

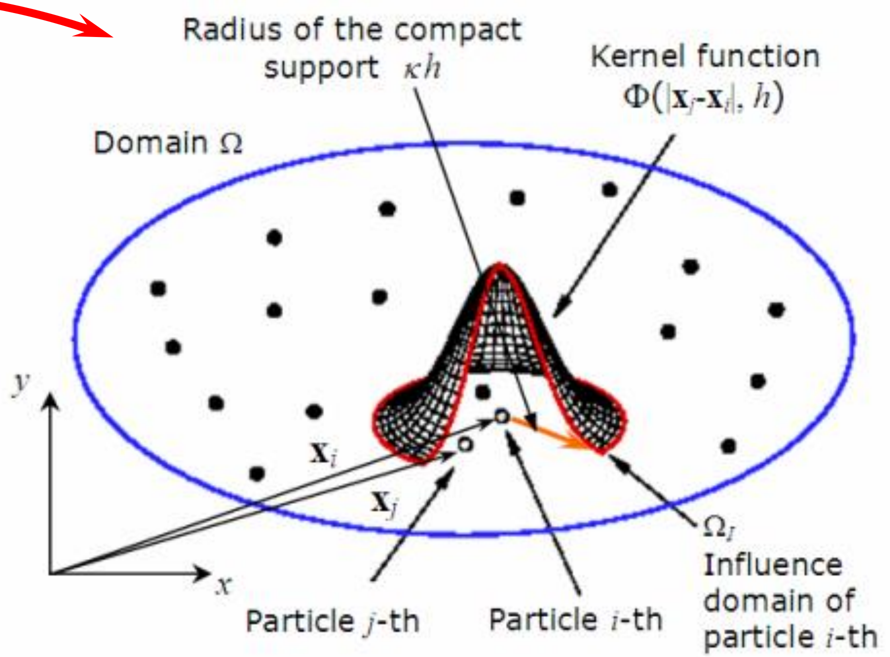
位移梯度场逼近

$$\nabla_{\mathbf{r}}\mathbf{u}(\mathbf{r})=\int W(\mathbf{r}-\mathbf{r}',h)\nabla_{\mathbf{r}'}\mathbf{u}(\mathbf{r}')\,\mathrm{d}\mathbf{r}'$$

$$=\int_{\partial} W(\mathbf{r}-\mathbf{r}',h)\mathbf{u}(\mathbf{r}')\cdot\mathbf{n}ds-\int\nabla_{\mathbf{r}'}W(\mathbf{r}-\mathbf{r}',h)\mathbf{u}(\mathbf{r}')\,\mathrm{d}\mathbf{r}'$$

$$=\int\nabla_{\mathbf{r}}W(\mathbf{r}-\mathbf{r}',h)\mathbf{u}(\mathbf{r}')\,\mathrm{d}\mathbf{r}'$$

$$\approx\sum_j\left[\nabla_{\mathbf{r}}W(\mathbf{r}-\mathbf{r}_j,h)\right]\mathbf{u}_j\frac{m_j}{\rho_j}$$



改进的位移梯度

$$\nabla_{\mathbf{r}}\mathbf{u}(\mathbf{r}) \approx \sum_j \left[\nabla_{\mathbf{r}} W(\mathbf{r}-\mathbf{r}_j, h)\right]\mathbf{u}_j \frac{m_j}{\rho_j}$$

结果不理想！

考虑： $\sum_j \left[\nabla_{\mathbf{r}} W(\mathbf{r}-\mathbf{r}_j, h)\right]\mathbf{u}\frac{m_j}{\rho_j} = \mathbf{u}\sum_j \left[\nabla_{\mathbf{r}} W(\mathbf{r}-\mathbf{r}_j, h)\right]\frac{m_j}{\rho_j} \approx \mathbf{u}\nabla_{\mathbf{r}}\mathbf{1} = 0$

$$\nabla_{\mathbf{r}}\mathbf{u}(\mathbf{r}) \approx \sum_j \left[\nabla_{\mathbf{r}} W(\mathbf{r}-\mathbf{r}_j, h)\right](\mathbf{u}_j - \mathbf{u})\frac{m_j}{\rho_j}$$

$$\nabla_{\mathbf{r}}\mathbf{u}_i \approx \sum_j \left[\nabla_{\mathbf{r}} W(\mathbf{r}_i-\mathbf{r}_j, h)\right](\mathbf{u}_j - \mathbf{u}_i)\frac{m_j}{\rho_j}$$

离散形式

应变逼近

$$\boldsymbol{\varepsilon}(\mathbf{r})=\frac{1}{2}\left[\nabla_{\mathbf{r}}\mathbf{u}(\mathbf{r})+\mathbf{u}(\mathbf{r})\nabla_{\mathbf{r}}\right]$$

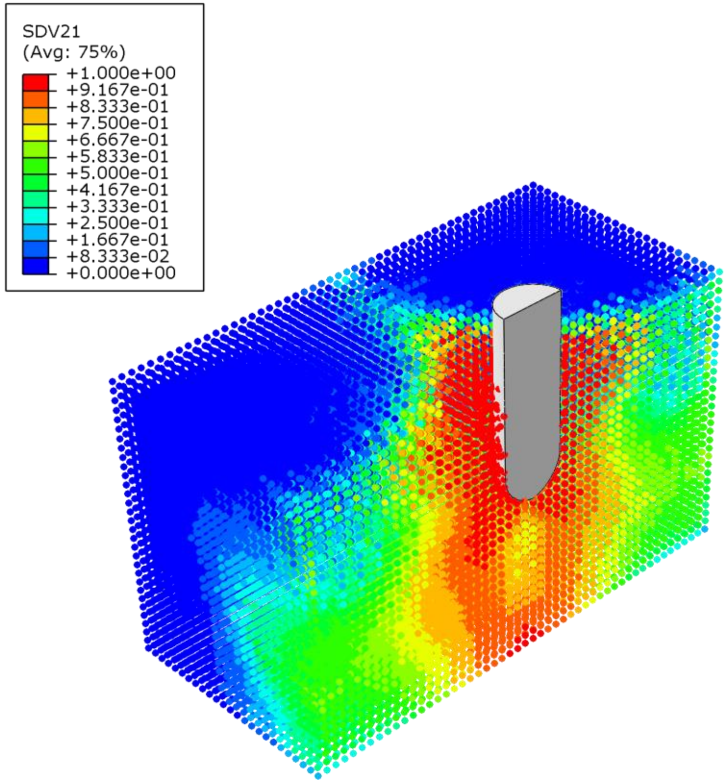
$$=\frac{1}{2}\sum_{j}\left[\nabla_{\mathbf{r}}W(\mathbf{r}-\mathbf{r}_j,h)\right](\mathbf{u}_j-\mathbf{u})\frac{m_j}{\rho_j}+\frac{1}{2}\sum_{j}\left[W(\mathbf{r}-\mathbf{r}_j,h)\nabla_{\mathbf{r}}\right](\mathbf{u}_j-\mathbf{u})\frac{m_j}{\rho_j}$$

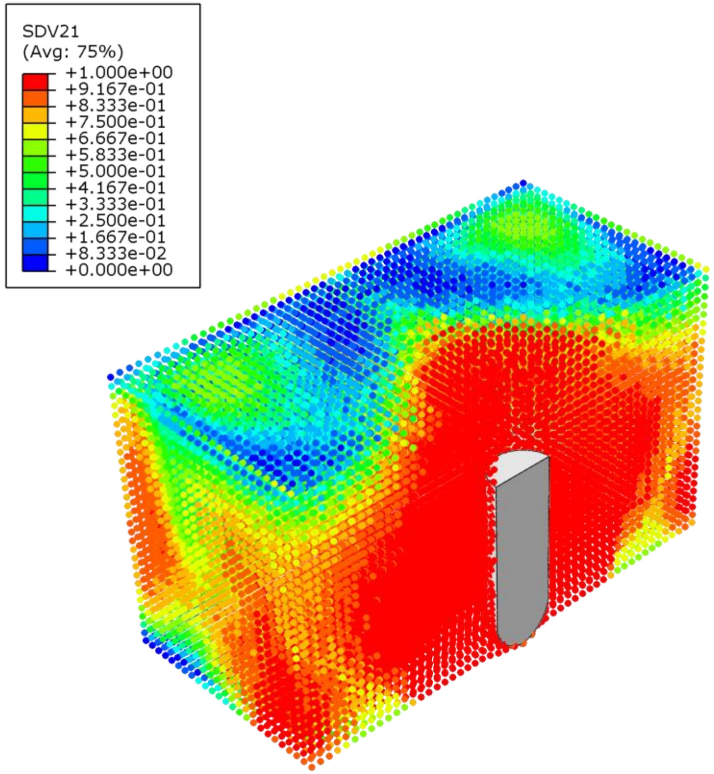
⇓ 代入控制方程的弱形式

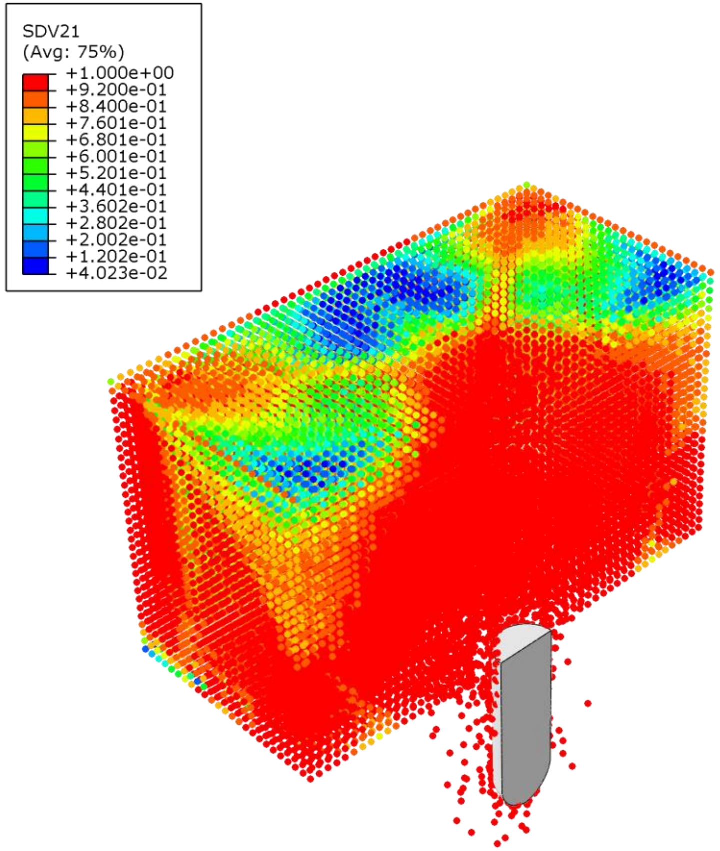
$$\int_{\Omega}\delta\boldsymbol{\varepsilon}:\boldsymbol{\sigma}d\Omega=\int_{\partial\Omega}\delta\mathbf{u}\cdot\mathbf{t}dS+\int_{\Omega}\delta\mathbf{u}\cdot\mathbf{b}d\Omega+\int_{\Omega}\rho\delta\mathbf{u}\cdot\ddot{\mathbf{u}}d\Omega$$

采用显式时域积分算法（绕过复杂的数值积分）

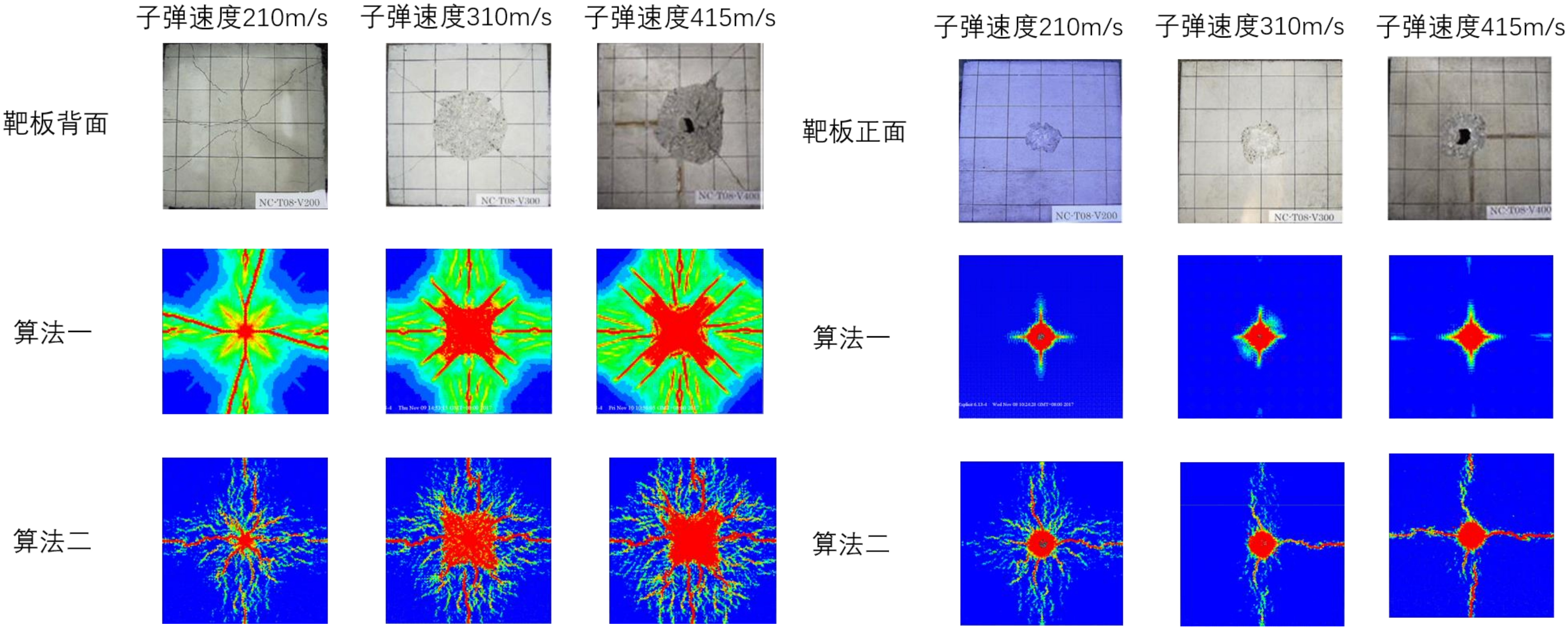
冲击贯穿问题


SDV21
(Avg: 75%)
+1.000e+00
+9.167e-01
+8.333e-01
+7.500e-01
+6.667e-01
+5.833e-01
+5.000e-01
+4.167e-01
+3.333e-01
+2.500e-01
+1.667e-01
+8.333e-02
+0.000e+00

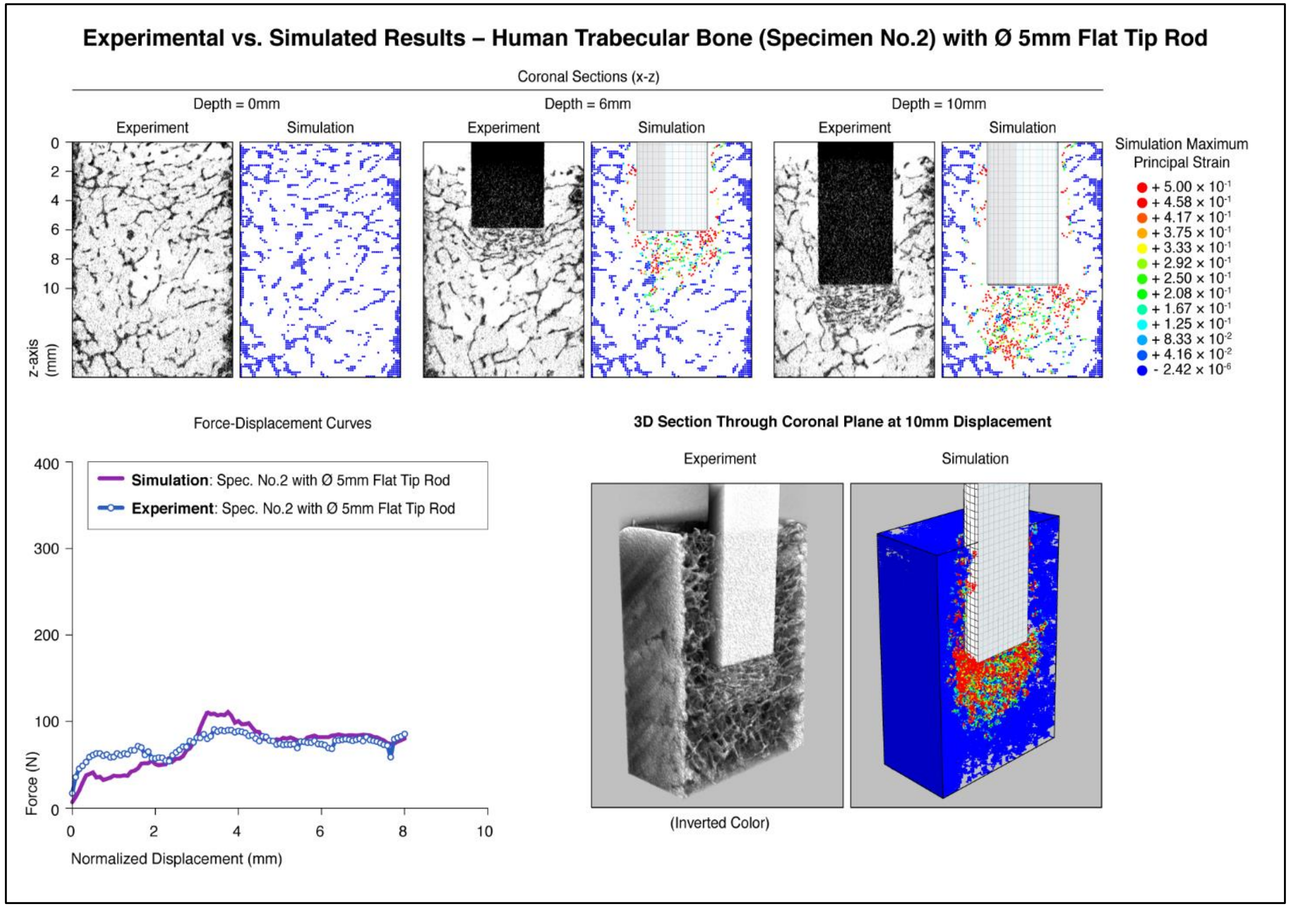

SDV21
(Avg: 75%)
+1.000e+00
+9.167e-01
+8.333e-01
+7.500e-01
+6.667e-01
+5.833e-01
+5.000e-01
+4.167e-01
+3.333e-01
+2.500e-01
+1.667e-01
+8.333e-02
+0.000e+00

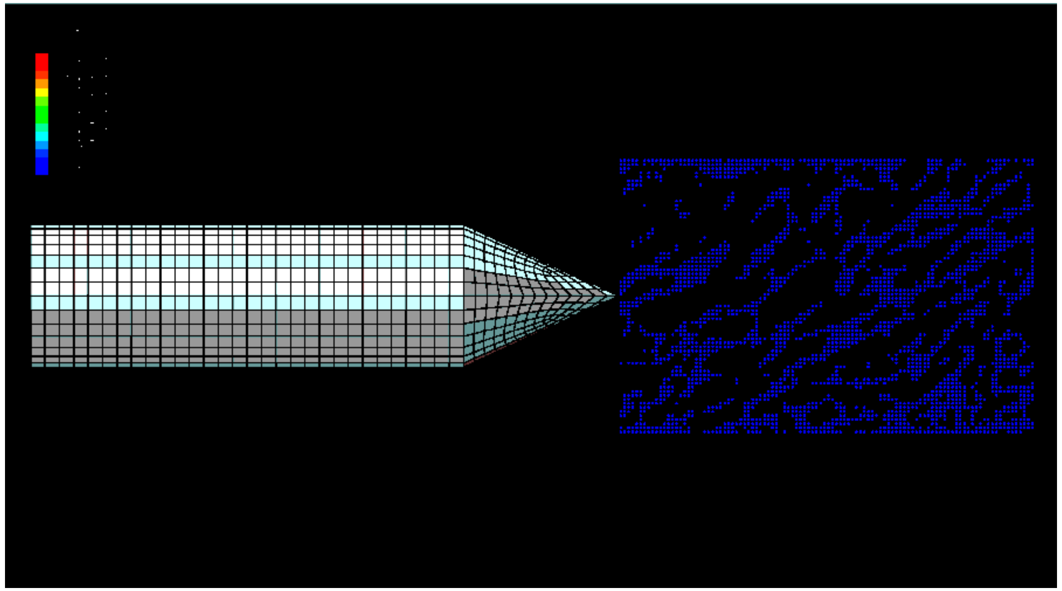

SDV21
(Avg: 75%)
+1.000e+00
+9.200e-01
+8.400e-01
+7.601e-01
+6.801e-01
+6.001e-01
+5.201e-01
+4.401e-01
+3.602e-01
+2.802e-01
+2.002e-01
+1.202e-01
+4.023e-02

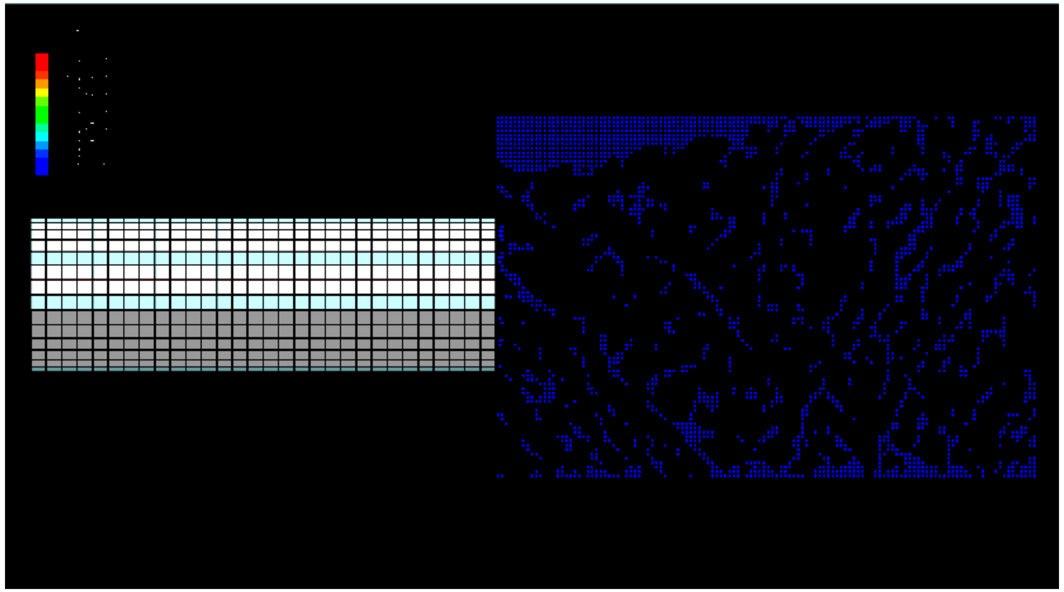
冲击贯穿问题


子弹速度210m/s
子弹速度310m/s
子弹速度415m/s
子弹速度210m/s
子弹速度310m/s
子弹速度415m/s
靶板背面
靶板正面
算法一
算法二
算法一
算法二
NC-T08-V200
NC-T08-V300
NC-T08-V200
NC-T08-V300

松质骨的锥入问题



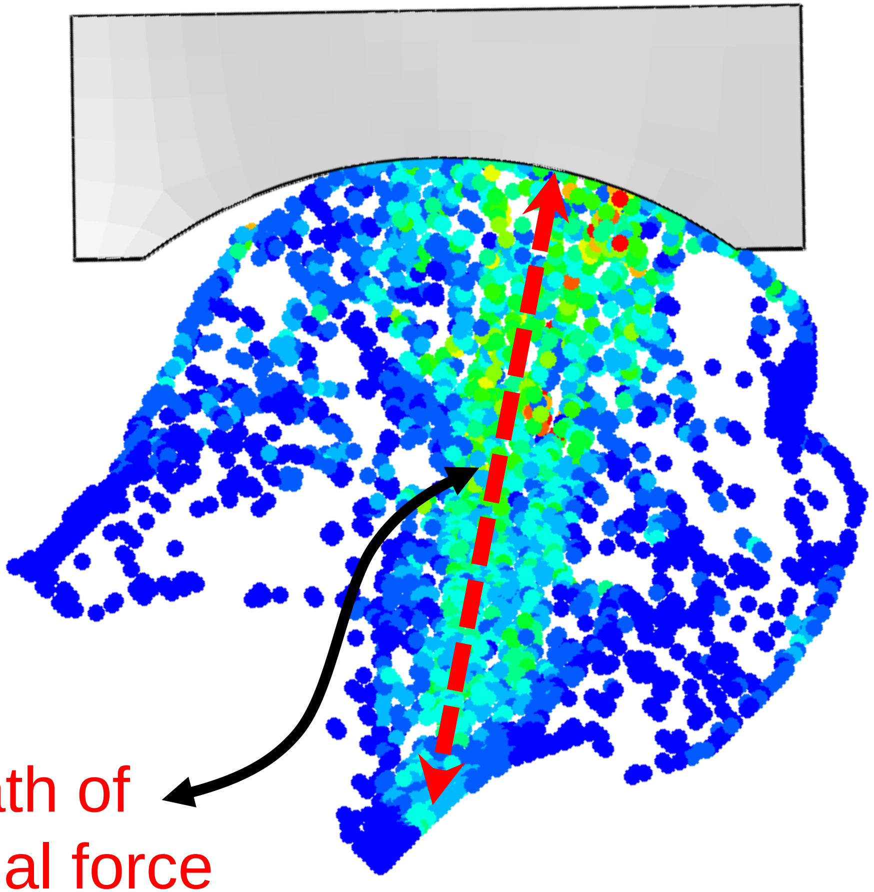




Kulper, S. A., Fang, C. X.*, Ren, X.*, Guo, M., Sze, K., Leung, F. K. and Lu, W. W. , Development and initial validation of a novel smoothed-particle hydrodynamics-based simulation model of trabecular bone penetration by metallic implants. J. Orthop. Res.. doi:10.1002/jor.23734

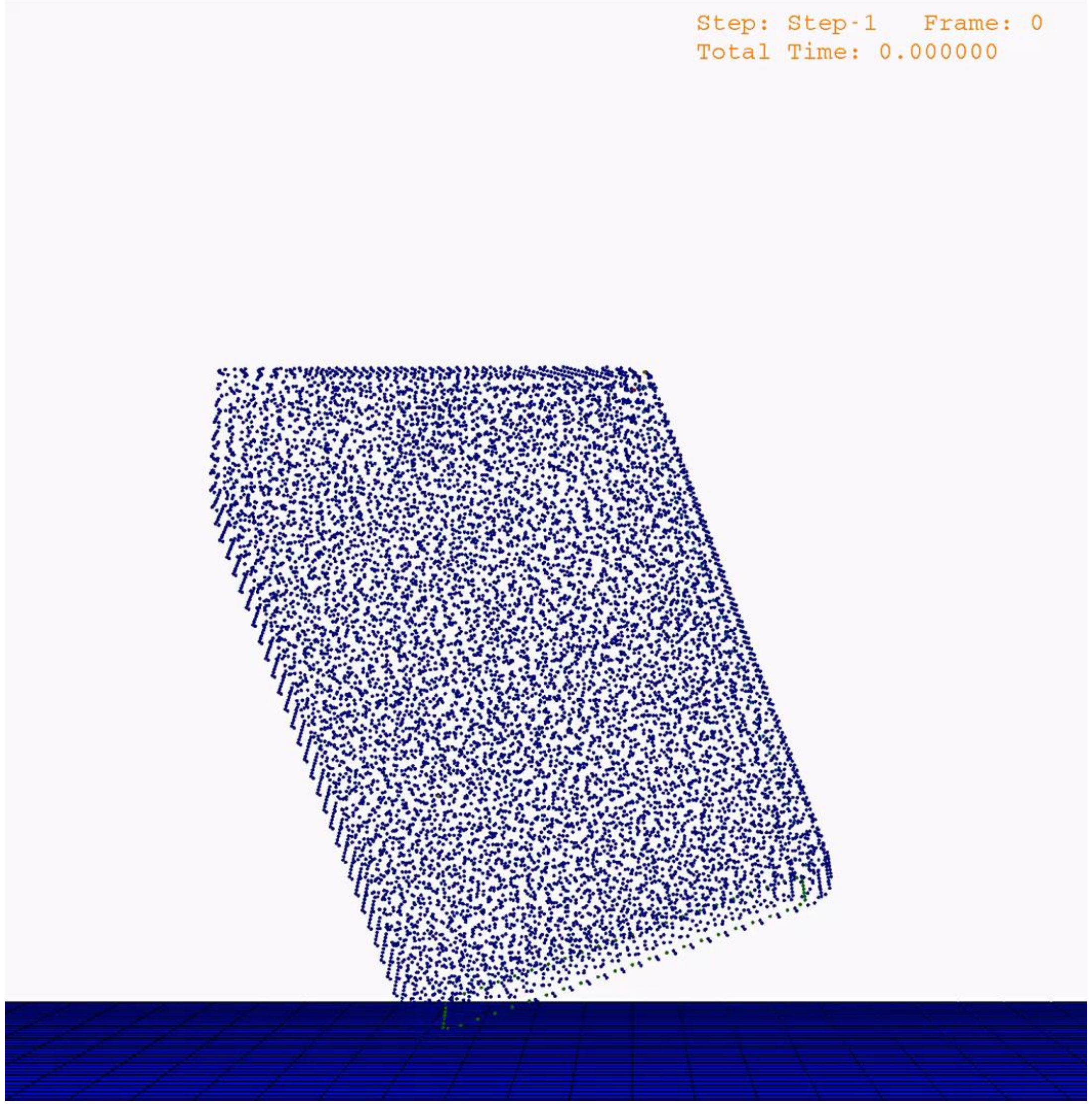
股骨头受力问题（**以下图片可能引起不适!!!**）

Experimental setup for a whole femur
with an metallic implant



流固耦合问题


Step: Step-1   Frame: 0
Total Time: 0.000000


Step: Step-1   Frame: 0
Total Time: 0.000000

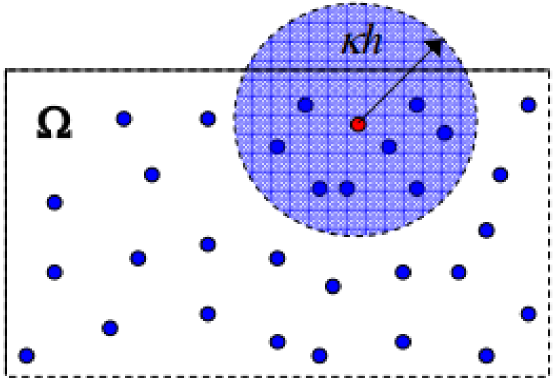
SPH的问题

□插值精度（连续性）不能保证

Tensile instability: causes exponential growth of particles velocity for small perturbation of their position in a region of the continuum with a tensile state of stress. When compression occurs clumping of particles may took place. Modified kernel function have been proposed for remedial [**Morris, 1996**].

不满足单位分解条件，不能精确再现常数场

$$f^k(\mathbf{x}) \approx \sum_{I=1}^{N} \frac{m_I W_a(\mathbf{x}-\mathbf{x}_I)}{\sum_{J=1}^{N} m_J W_a(\mathbf{x}_I-\mathbf{x}_J)} \neq 1$$



Reproducing Kernel Particle Method

REPRODUCING KERNEL PARTICLE METHOD (RKPM)

$$u^h(\mathbf{x}) = \sum_I \phi_I(\mathbf{x}) u_I$$

$$\phi_I(\mathbf{x}) = \varphi_a(\mathbf{x} - \mathbf{x}_I) \left(\sum_{|\alpha| \le n} (\mathbf{x} - \mathbf{x}_I)^\alpha b_\alpha(\mathbf{x}) \right)$$

REPRODUCING CONDITION

$$\sum_I \phi_I(\mathbf{x})\mathbf{x}_I{}^{\alpha} = \mathbf{x}^{\alpha}, \quad |\alpha| \le n$$

$$\phi_I(\mathbf{x}) = \mathbf{H}^T(\mathbf{0})\mathbf{M}^{-1}(\mathbf{x})\mathbf{H}^T(\mathbf{x}-\mathbf{x}_I)\varphi_a(\mathbf{x}-\mathbf{x}_I)$$

$$\mathbf{H}(\mathbf{x}) = \{\mathbf{x}^{\alpha}\}_{|\alpha|\le n} = \{1, x_1, \cdots, x_d^n\}$$

$$\mathbf{M}(\mathbf{x}) = \sum_I \mathbf{H}(\mathbf{x}-\mathbf{x}_I)\mathbf{H}^T(\mathbf{x}-\mathbf{x}_I)\varphi_a(\mathbf{x}-\mathbf{x}_I)$$

代入控制方程的弱形式

$$u^h(\mathbf{x}) = \sum_I \phi_I(\mathbf{x}) u_I$$

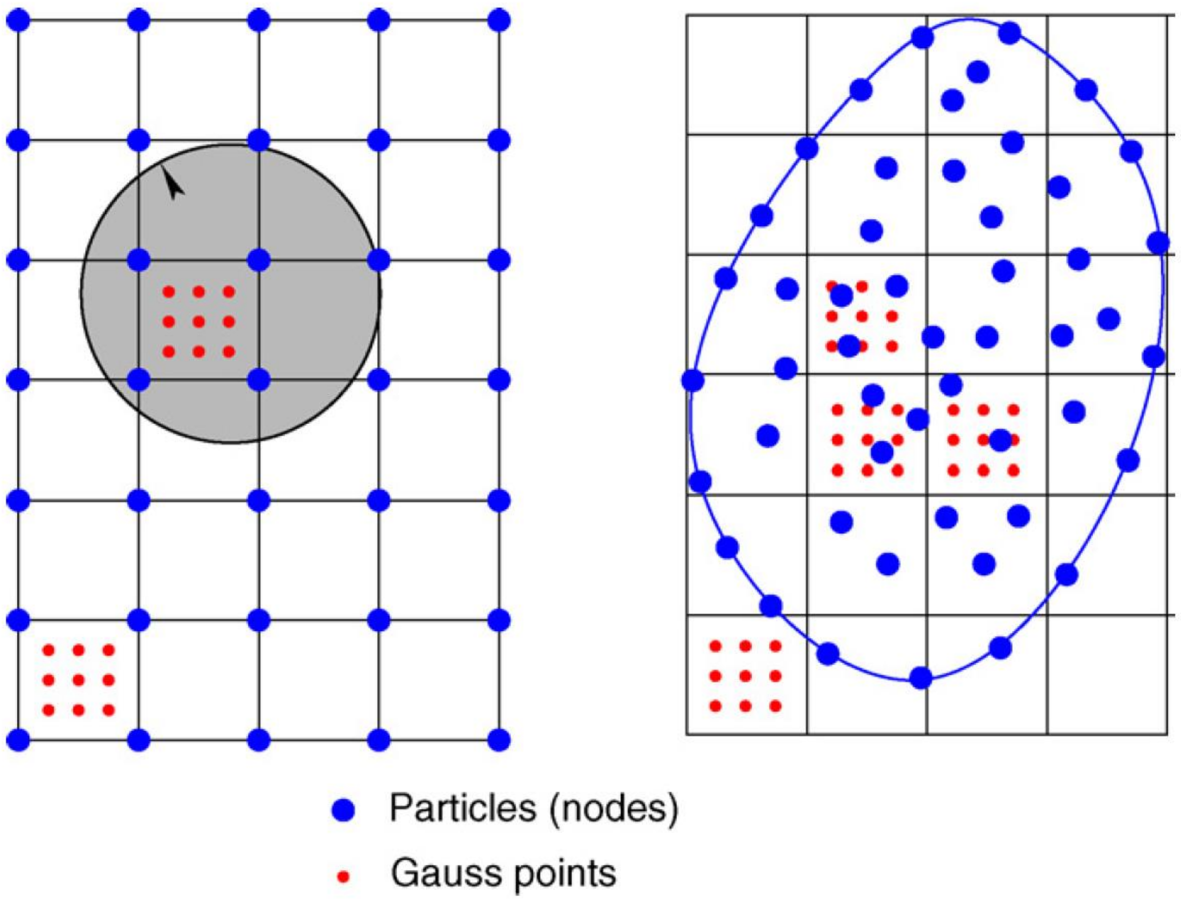
⇩

$$\int_\Omega \delta\boldsymbol{\varepsilon} : \boldsymbol{\sigma} d\Omega = \int_{\partial\Omega} \delta\mathbf{u} \cdot \mathbf{t} dS + \int_\Omega \delta\mathbf{u} \cdot \mathbf{b} d\Omega + \int_\Omega \rho \delta\mathbf{u} \cdot \ddot{\mathbf{u}} d\Omega$$

数值积分？？？

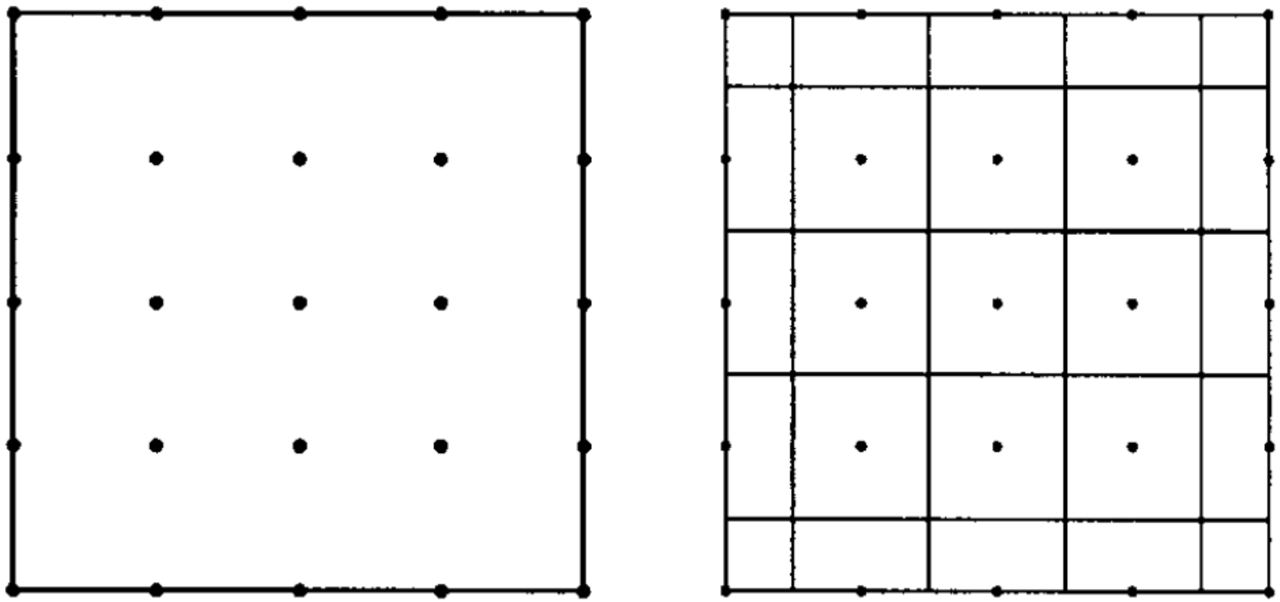
数值积分

基于背景网格的数值积分



Background mesh **Background cell**

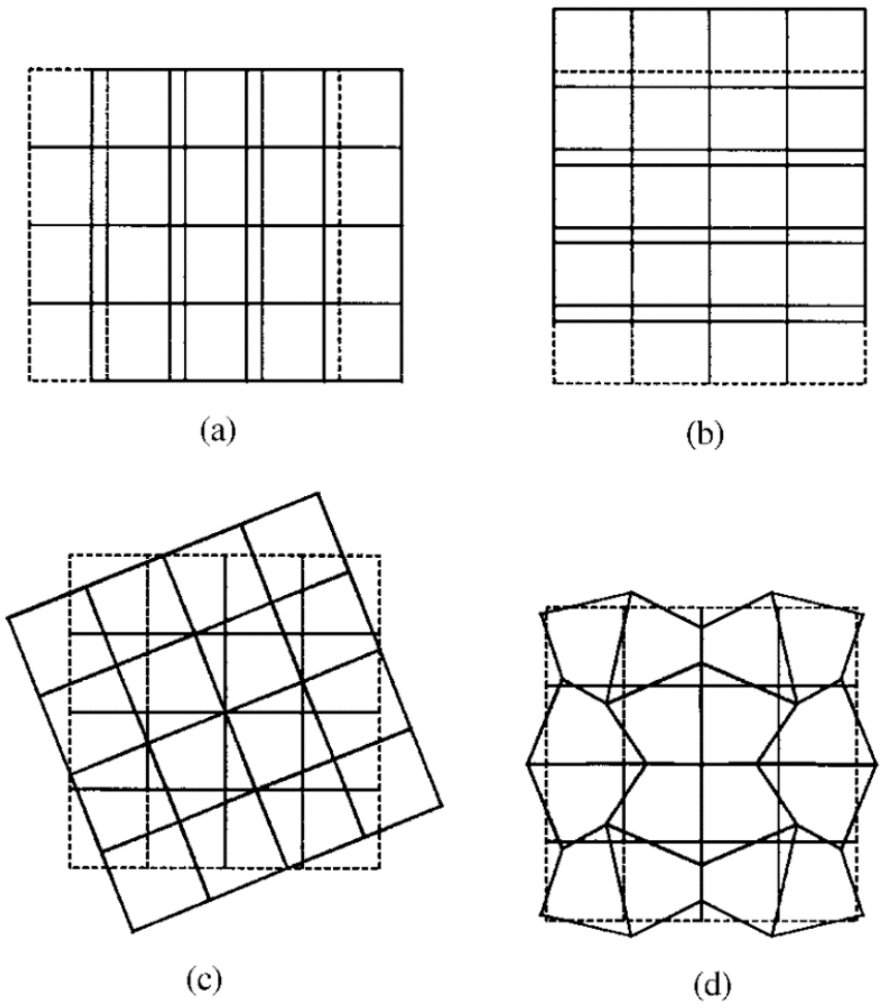
节点积分（NODAL INTEGRATION）



(a) (b)

Discretization and nodal representative domain for nodal integration: (a) mesh-free discretization; and (b) nodal representative domain.

节点积分（NODAL INTEGRATION）



Spurious zero energy mode !

稳定节点积分（STABILIZED CONFORMING NODAL INTEGRATION）

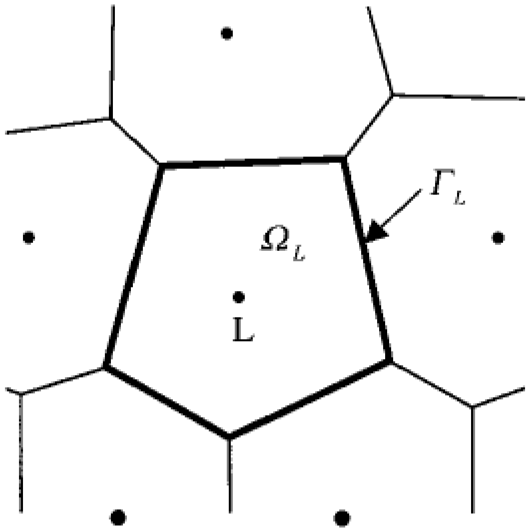
$$\mathbf{K}=\int_{\Omega}\mathbf{B}^{T}\mathbf{D}\mathbf{B}d\Omega=\sum_{L}\mathbf{B}_{L}^{T}\mathbf{D}_{L}\mathbf{B}_{L}A_{L}$$

节点积分

$$\bar{\varepsilon}_{ij}(\mathbf{x}_{L})=\int_{\Omega}\tilde{\varepsilon}_{ij}(\mathbf{x}_{L})\Phi(\mathbf{x}_{L};\mathbf{x}-\mathbf{x}_{L})dx$$

$$\bar{\varepsilon}_{ij}(\mathbf{x}_{L})=\int_{\Omega}\frac{1}{2}\; u_{i,j}+u_{j,i}\;\;\Phi(\mathbf{x}_{L};\mathbf{x}-\mathbf{x}_{L})dx$$

$$=\frac{1}{2A_{L}}\int_{\Gamma_{L}}(u_{i}n_{j}+u_{j}n_{i})d\Gamma$$

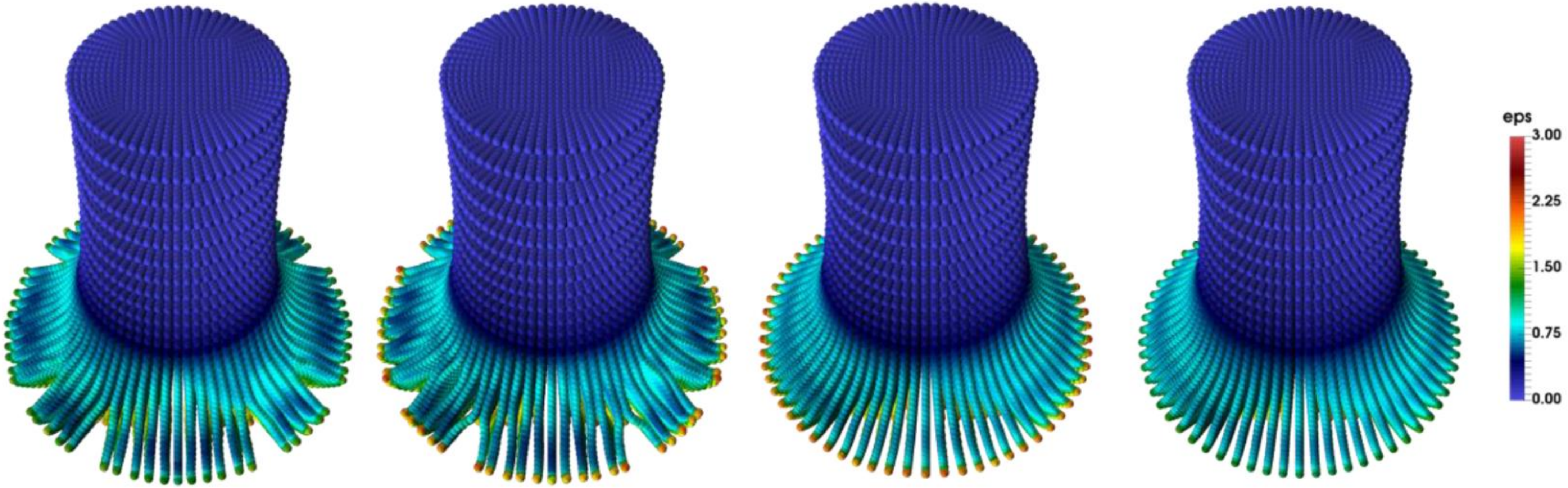


$$\Phi(\mathbf{x}_{L};\mathbf{x}-\mathbf{x}_{L})=\begin{cases}1/A_{L} & \mathbf{x}\in\Omega_{L}\\ 0 & \mathbf{x}\notin\Omega_{L}\end{cases}$$

$$\tilde{\mathbf{B}}_{I}(\mathbf{x}_{L})=\begin{bmatrix}\tilde{b}_{I1}(\mathbf{x}_{L}) & 0\\ 0 & \tilde{b}_{I2}(\mathbf{x}_{L})\\ \tilde{b}_{I2}(\mathbf{x}_{L}) & \tilde{b}_{I1}(\mathbf{x}_{L})\end{bmatrix}\qquad \tilde{b}_{Ii}(\mathbf{x}_{L})=\frac{1}{A_{L}}\int_{\Gamma_{L}}N_{I}(\mathbf{x})n_{i}(\mathbf{x})d\Gamma$$

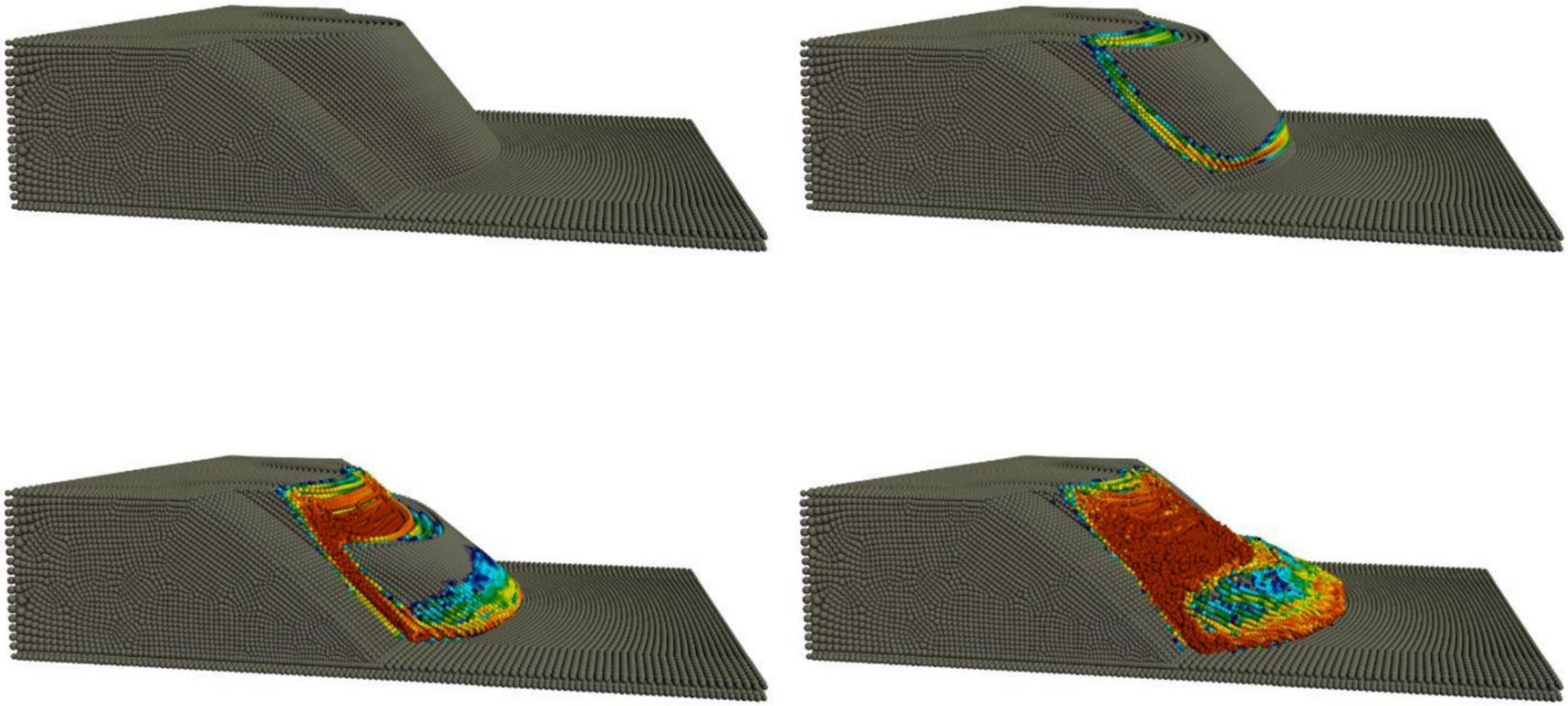
Chen, J. S., Wu, C. T., Yoon, S., and You, Y., "A Stabilized Conforming Nodal Integration for Galerkin Meshfree Methods," IJNME, Vol. 50, pp. 435-466, 2001.

TAYLOR BAR

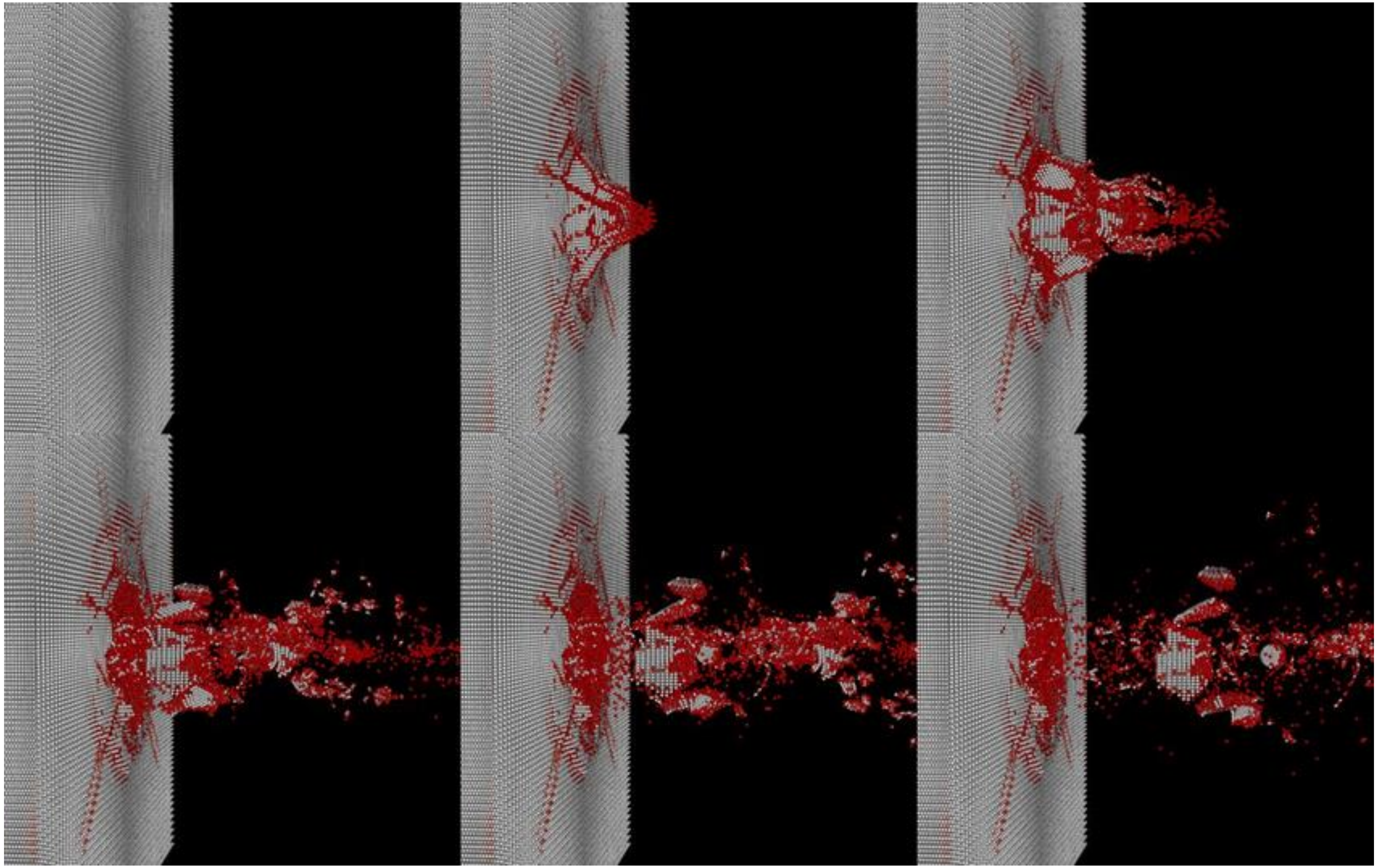


直接节点积分　　　　稳定节点积分

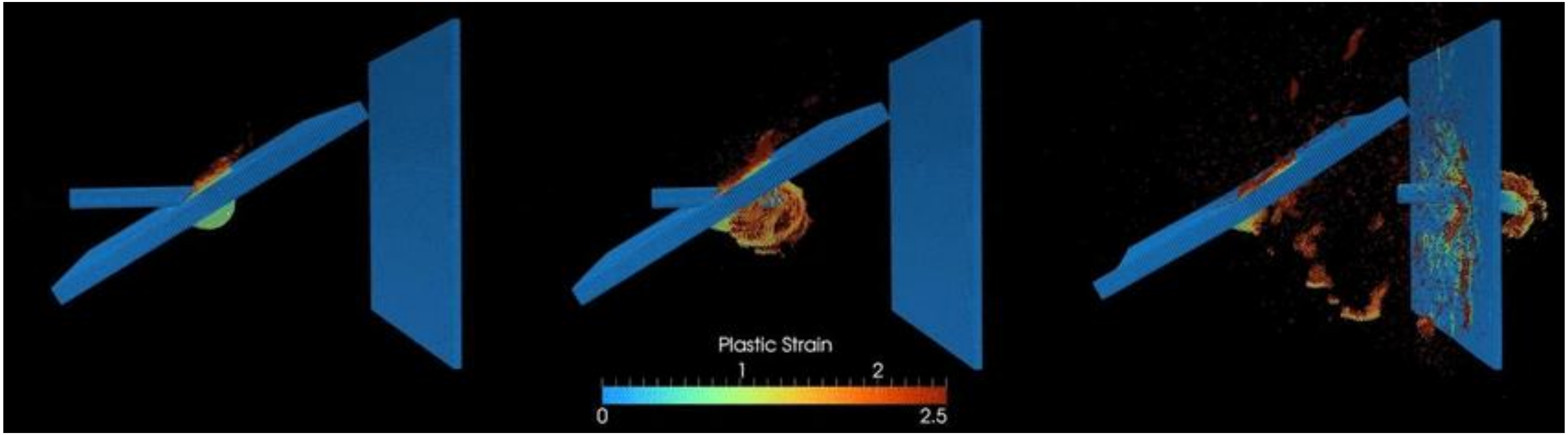
滑坡问题



混凝土板贯穿问题



多重板冲击问题

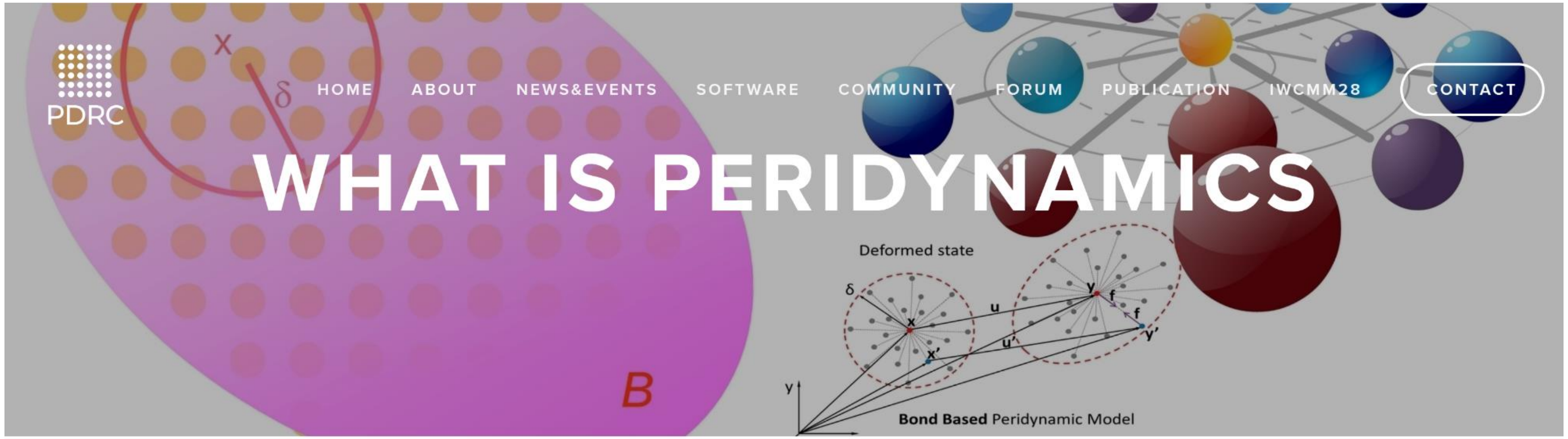


Peridynamics（近场/毗域动力学）

PERIDYNAMICS（近场/毗域动力学）

Prefix: peri

Around or surrounding: perimeter; near: perinatal



About

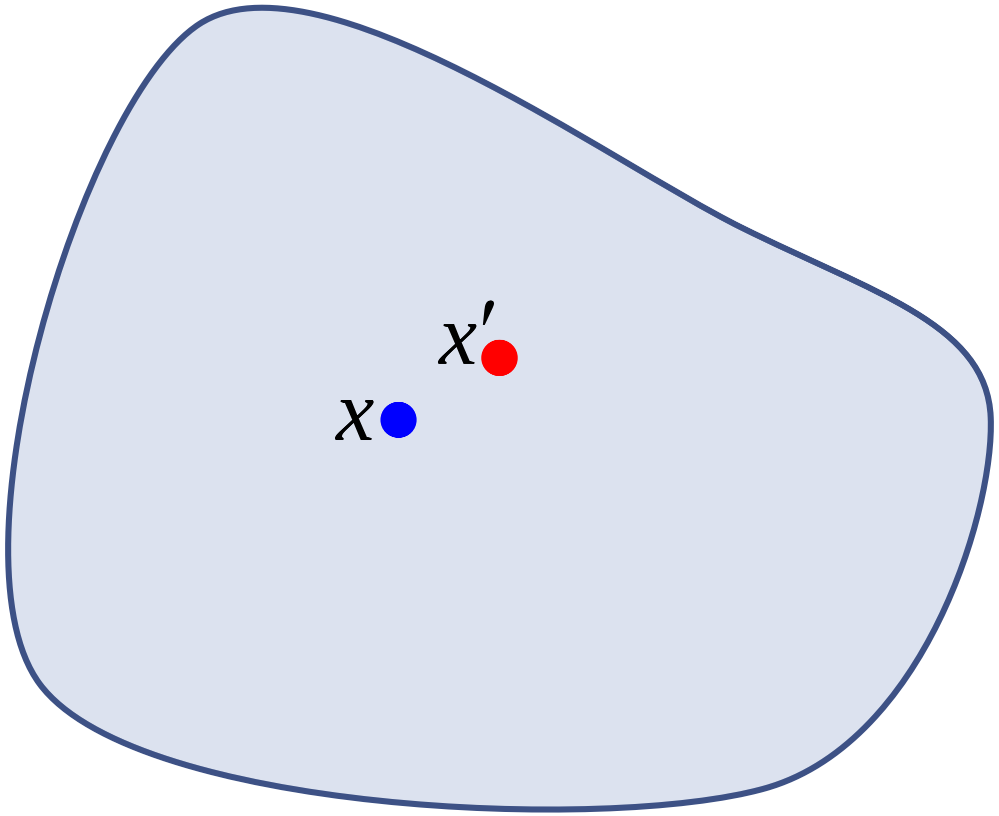
WHAT IS PERIDYNAMICS

DR. STEWART SILLING

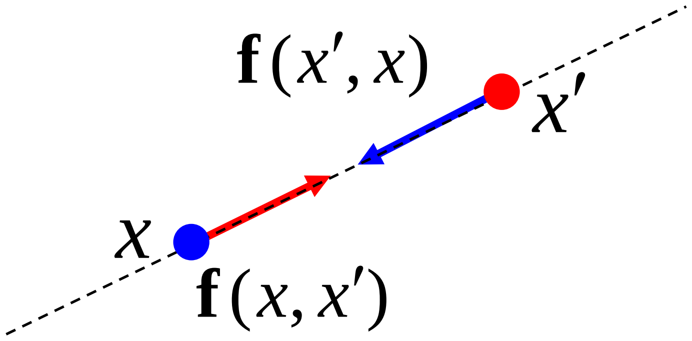
Peridynamics is a new continuum mechanics formulation. It was originally developed by Dr. Stewart Silling in 2000.

Stewart Silling

我一直在想的一个问题。。。



x与x'之间的内力是怎样的传递的？

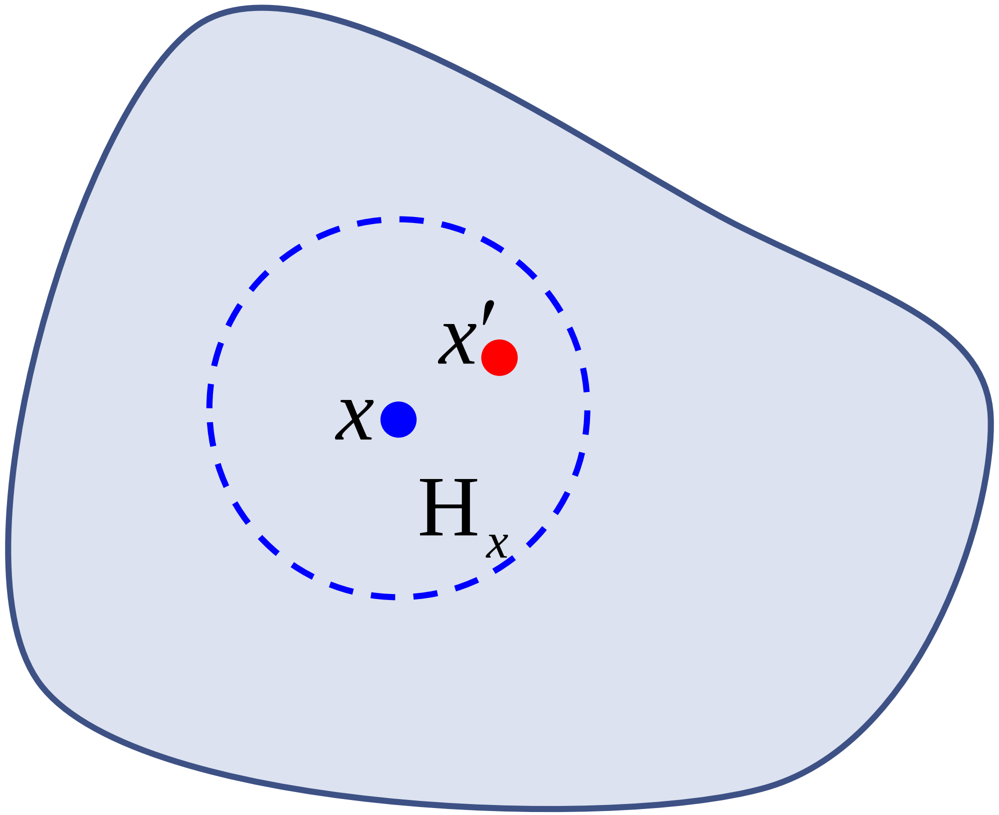


比如：沿着两个点的连线传递

键基近场动力学

Bond-based peridynamics

键基近场动力学



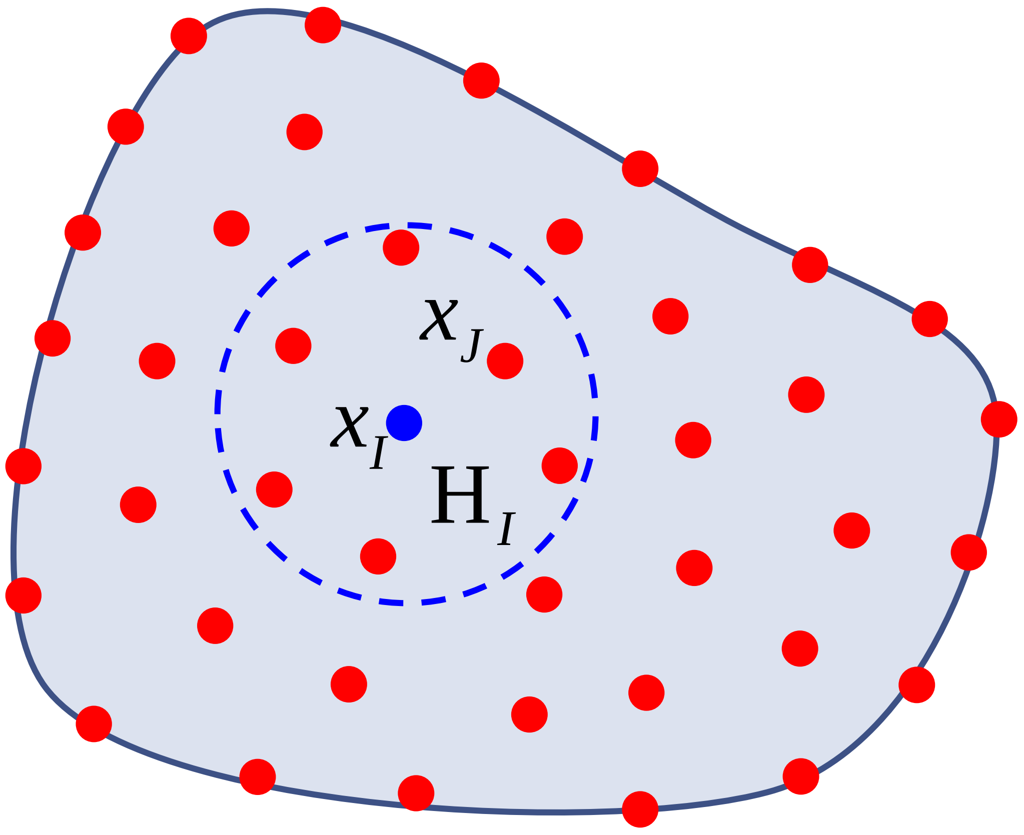
点x的内力合力：

$$\mathbf{L}(x,t)=\int_{H_x}\mathbf{f}(x,x')\mathrm{d}V_{x'}$$

点x的平衡方程：

$$\rho\ddot{\mathbf{u}}(x,t)=\mathbf{L}(x,t)+\mathbf{b}(x,t)$$

键基近场动力学



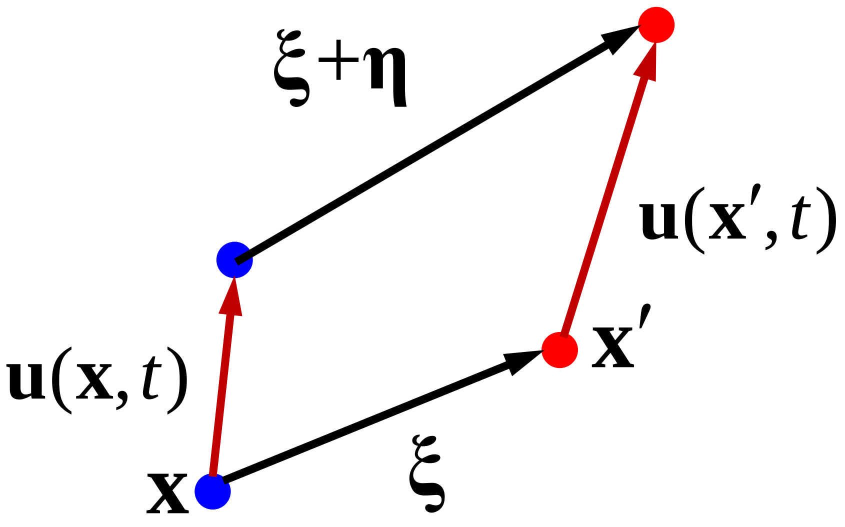
控制方程的离散形式：

$$\mathbf{L}(x_I,t)=\sum_{J\in H_I}\mathbf{f}(x_I,x_J)V_J$$

$$\rho\ddot{\mathbf{u}}(x_I,t)=\mathbf{L}(x_I,t)+\mathbf{b}(x_I,t)$$

采用中心差分法等显式时域积分方法求解。

键基近场动力学



$$\boldsymbol{\xi} = \mathbf{x}' - \mathbf{x}$$

$$\boldsymbol{\eta} = \mathbf{u}' - \mathbf{u}$$

轴向拉压应变： $s = \dfrac{\|\boldsymbol{\xi}+\boldsymbol{\eta}\| - \|\boldsymbol{\xi}\|}{\|\boldsymbol{\xi}\|}$

应变能： $w(\boldsymbol{\xi},\boldsymbol{\eta}) = \dfrac{1}{2} c s^2 \|\boldsymbol{\xi}\|$

内力： $\mathbf{f}(\boldsymbol{\xi},\boldsymbol{\eta}) = \dfrac{\partial w}{\partial \boldsymbol{\eta}}$

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$$\mathbf{f}(\boldsymbol{\xi},\boldsymbol{\eta})=\frac{\partial w}{\partial \boldsymbol{\eta}}=\frac{\partial w}{\partial s}\frac{\partial s}{\partial \boldsymbol{\eta}}$$

$$w(\boldsymbol{\xi},\boldsymbol{\eta})=\frac{1}{2}cs^2\|\boldsymbol{\xi}\| \quad s=\frac{\|\boldsymbol{\xi}+\boldsymbol{\eta}\|-\|\boldsymbol{\xi}\|}{\|\boldsymbol{\xi}\|}$$

$$\frac{\partial w}{\partial s}=cs\|\boldsymbol{\xi}\|$$

$$\frac{\partial s}{\partial \boldsymbol{\eta}}=\frac{\boldsymbol{\xi}+\boldsymbol{\eta}}{\|\boldsymbol{\xi}+\boldsymbol{\eta}\|\|\boldsymbol{\xi}\|}$$

$$\mathbf{f}(\boldsymbol{\xi},\boldsymbol{\eta})=cs\frac{\boldsymbol{\xi}+\boldsymbol{\eta}}{\|\boldsymbol{\xi}+\boldsymbol{\eta}\|} \qquad \mathbf{n}=\frac{\boldsymbol{\xi}+\boldsymbol{\eta}}{\|\boldsymbol{\xi}+\boldsymbol{\eta}\|}$$

考虑Horizon，内力幅值：

$$f=\begin{cases}0 & \|\boldsymbol{\xi}\|>\delta\\ cs\mu & \|\boldsymbol{\xi}\|\le\delta\end{cases}$$

考虑键的断裂，引入函数：

$$\mu=\begin{cases}1 & s(\boldsymbol{\xi},t)\le s_0\\ 0 & otherwise\end{cases}$$

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对于一维弹性杆问题：

$$W=\frac{1}{2}\int_{\mathrm{H}}w(\boldsymbol{\xi},\boldsymbol{\eta})\mathrm{d}V_{\xi}=\frac{A}{2}\int_{-\delta}^{\delta}w(\boldsymbol{\xi},\boldsymbol{\eta})\mathrm{d}\xi=\frac{A}{2}\int_{-\delta}^{\delta}\frac{1}{2}cs^{2}\xi\mathrm{d}\xi=\frac{A}{4}c_{0}s^{2}\delta^{2}$$

$$W=\frac{1}{2}Es^{2}\quad\Rightarrow\quad c_{0}=\frac{2E}{A\delta^{2}}$$

对于不同维度的问题，不同的Horizon范围，上述关系是不一样。等效泊松比是一般也是固定值。

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对于断裂问题，断裂阈值

$$s_0 = \frac{10G_0}{\pi c \delta^5}$$

材料断裂能

For 3D : $G_0 = \frac{1}{2} c s_0^2 \left(\frac{\pi \delta^5}{5} \right)$

$$G_0 = \frac{1}{2} c s_0^2 \left(\frac{B\delta^4}{2} \right)$$

$$\cos\theta = \frac{z}{\xi}$$

$$G_0 = 2 \int_0^{\delta} \int_z^{\delta} \int_0^{\cos^{-1}\left(\frac{z}{\xi}\right)} \left[\frac{c(\xi)\, s_0^2 \xi}{2} \right] \xi \, d\theta \, d\xi \, dz$$

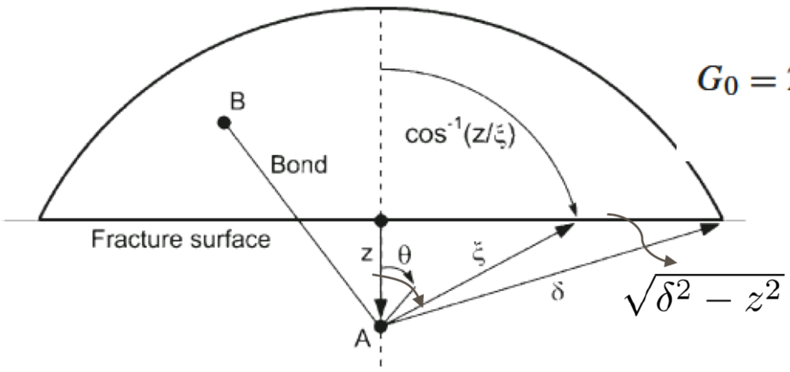
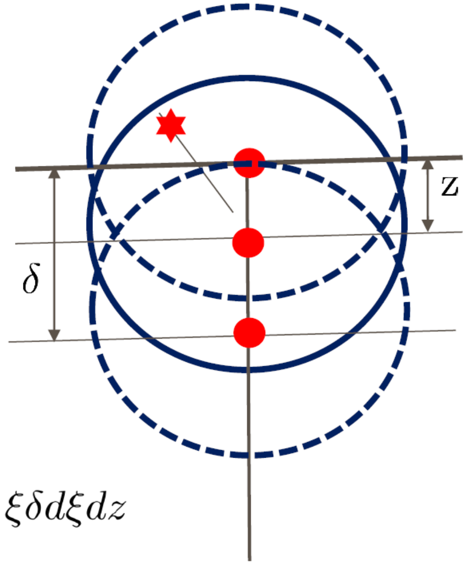


Fig. 3 Evaluation of fracture energy. For each point *A* along the *dashed line*, $0 \leq z \leq \delta$, the work required to break the bonds connecting *A* to each point *B* in the circular cap is given by Eq. (8)



$$G_0 = 2 \int_0^{\delta} \frac{1}{\sqrt{\delta^2 - z^2}} \int_z^{\delta} \int_0^{\delta \cos^{-1}(z/\xi)} \left(\frac{c(\xi) s_0^2 \xi}{2} \right) \xi \delta \, d\xi \, dz$$

Silling and Askari, 2005. A meshfreemethod based on the peridynamicmodel of solid mechanics. Computers and Structures 83 (2005) 1526-1535.

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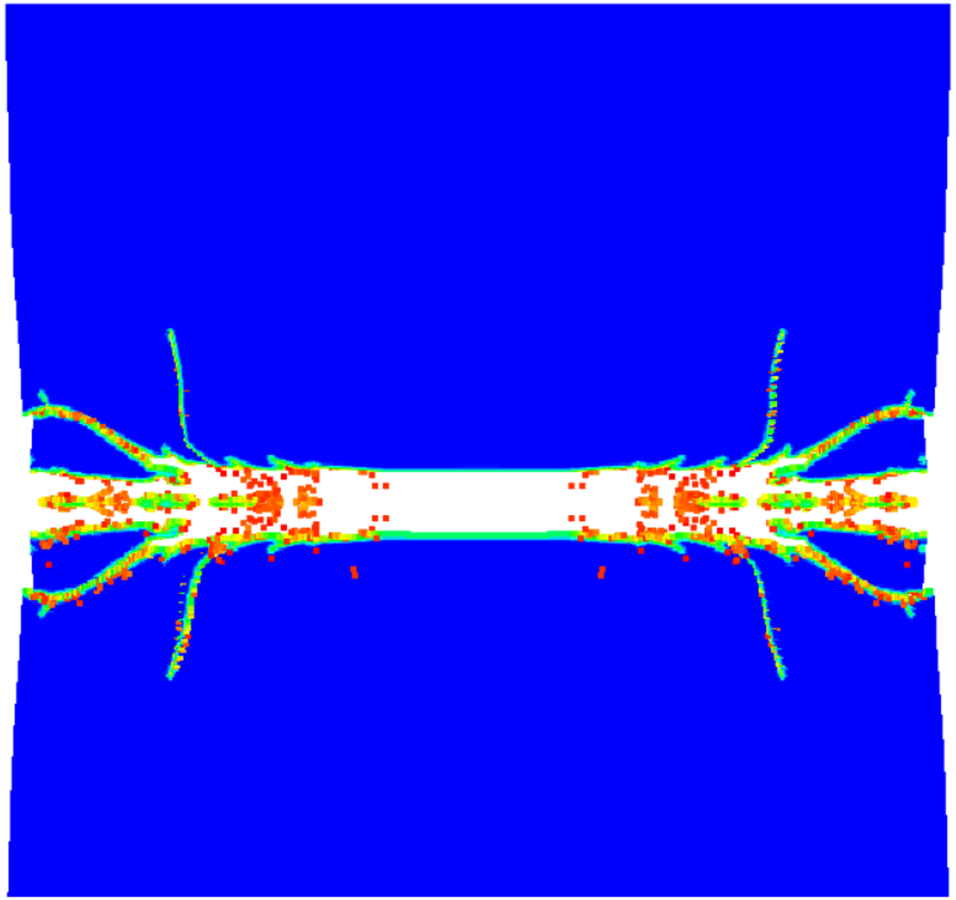

李少凡教授示例程序结果

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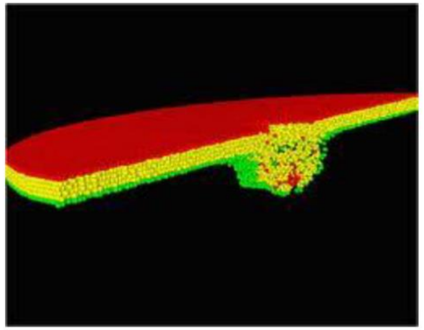

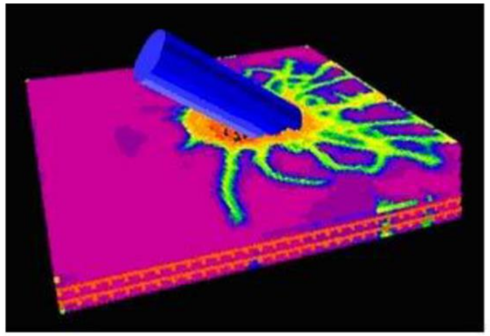

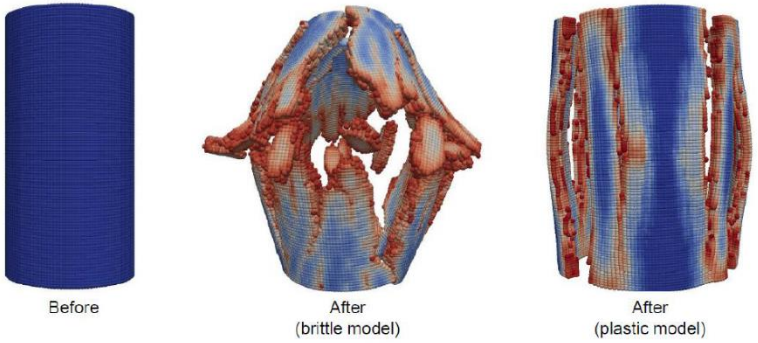

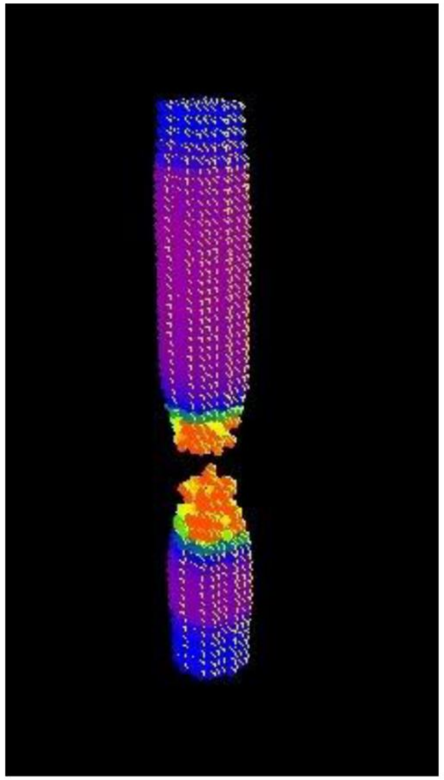

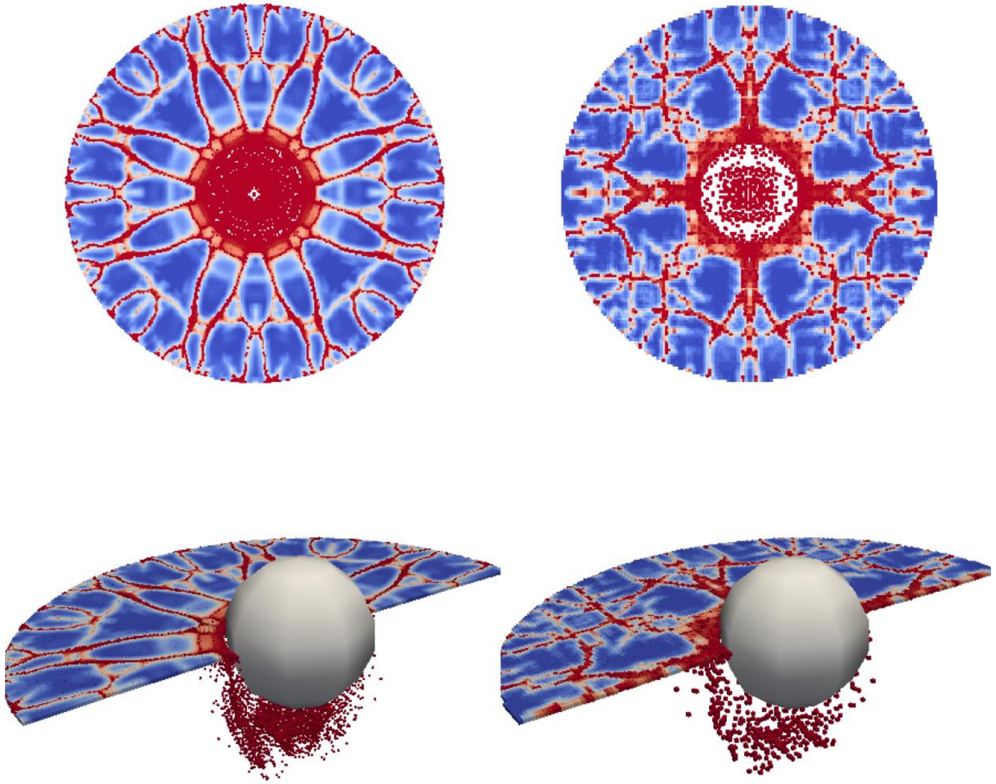

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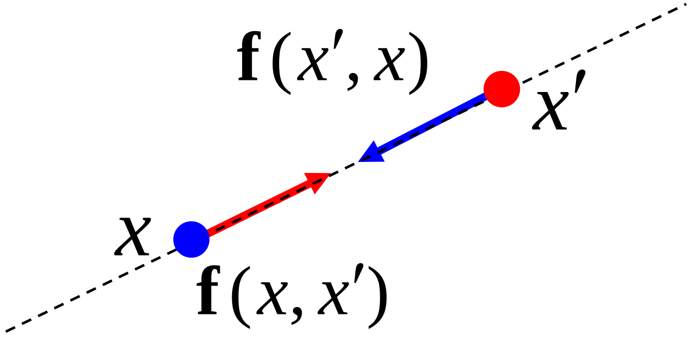
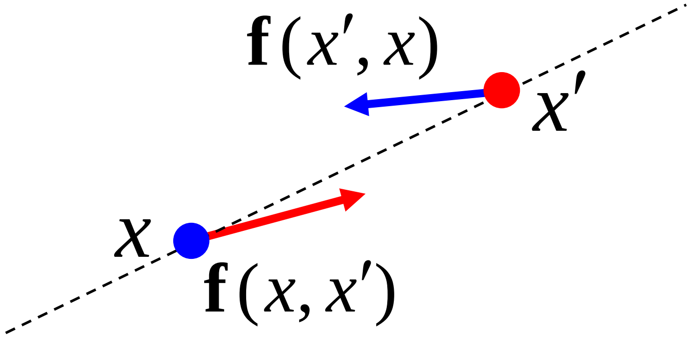
优点：简单粗暴！

缺点：过于简单粗暴！

回到原来的问题：x与x’之间的内力是怎样的传递的？





比如：沿着两个点的连线传递

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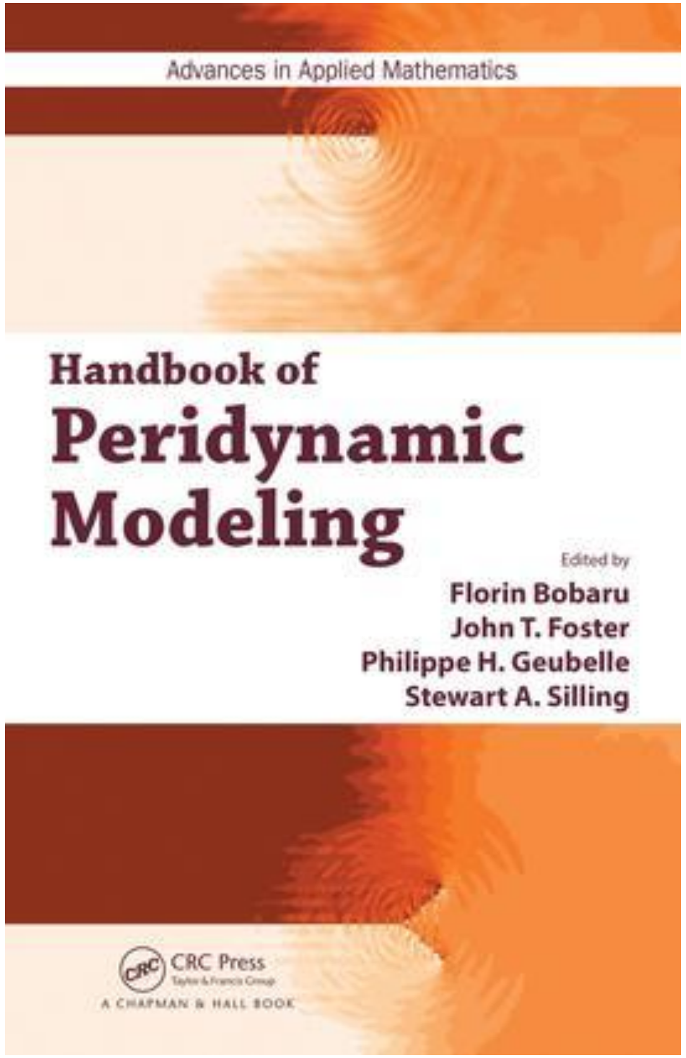
Bond-based peridynamics

比如：不沿着两个点的连线传递

态基近场动力学

State-based peridynamics

态基近场动力学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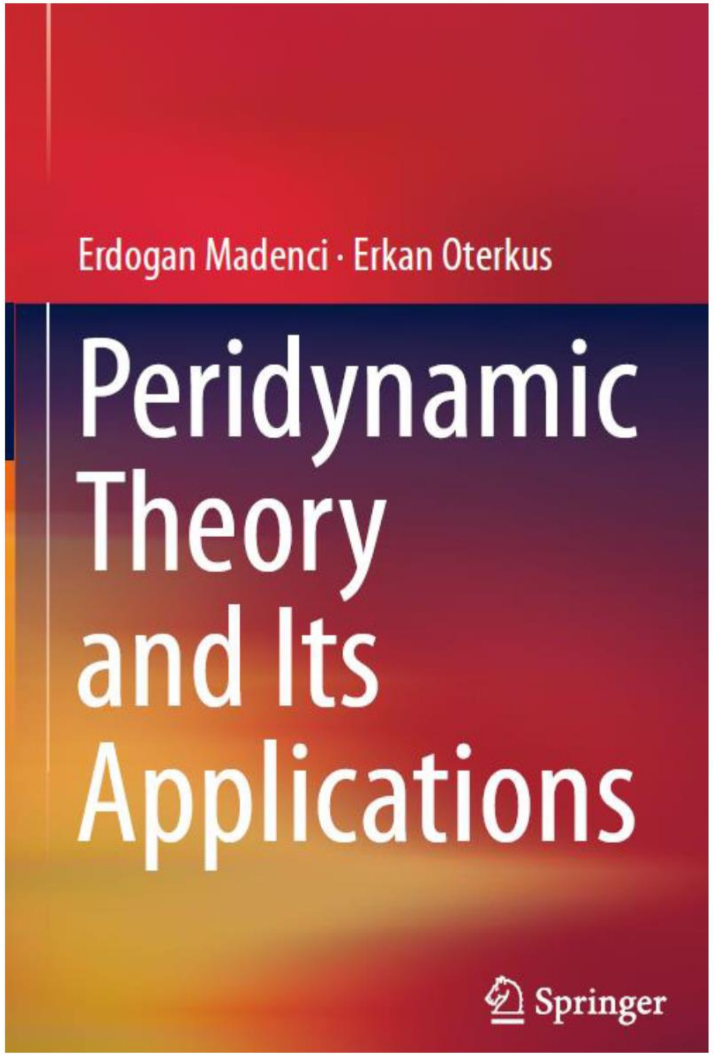
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