Uniaxial Elasto-Plastic Damage Model for Concrete Based on Chinese Design Code

Code Developed by

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1. Theoretical Framework

The uniaxial elasto-plastic damage model for concrete is based on the work by Wu *et al,* (2006), Ren (2010), Zeng (2012) and the Chinese design code (Ministry of Construction of the People's Republic of China, 2010). The stress-strain relation is shown in Fig. 1, the tensile and the compressive stress-strain behavior can be expressed as follows:

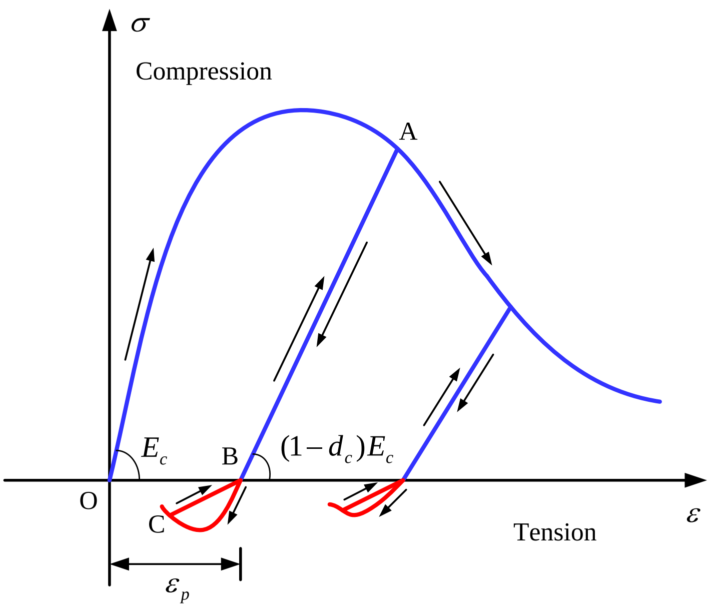


Fig. 1. Stress-strain relationship

Curve OA, monotonic compressive loading:

$$\sigma = (1-d_c^{'})E_c(\varepsilon_c - \varepsilon_p) \tag{1}$$

Curve AB, compressive unloading and reloading:

$$\sigma = (1 - d_c^{'})E_c(\varepsilon_c - \varepsilon_p) \tag{2}$$

Curve BC, monotonic tensile loading:

$$\sigma = (1 - d_{tc})E_c(\varepsilon_t - \varepsilon_p) \tag{3}$$

Curve CB, tensile unloading and reloading:

$$\sigma = (1 - d_{tc})E_c(\varepsilon_t - \varepsilon_p) \tag{4}$$

where σ = uniaxial stress; E_c = initial elastic modulus; ε_c = the compressive strain; ε_t = tensile strain; $d_c^{'}$ =elasto-plastic compressive damage variable; d_{tc} =modified tensile damage; ε_p = plastic strain. Considering that tensile plastic stain is usually small, therefore tensile plastic stain is set as zero and the total plastic strain is only related to the compression and can be computed by

$$\varepsilon^p = \phi_p \max(\varepsilon_c) = \xi_p \left[\exp(n_p d_c) - 1\right] \max(\varepsilon_c) \tag{5}$$

where ϕ_p is the plastic strain coefficient; ξ_p and n_p are parameters with values that range from 0.2 to 0.3 and 1.0 to 1.3, respectively (Ren, 2010, Zeng, 2012).

It should be noted that the compressive stress and strain are set as positive while the tensile stress and strain are set as negative herein for convenience of discussion. Equations (1) through (4), although look similar, have different meanings. In the monotonic stages the damage variables, $d_c^{'}$ and d_{tc}, change according to the loadings process so that the stress-strain relation is nonlinear. On the other hand, during unloading and reloading, the damage variables are constant resulting in a linear behavior.

The damage evolution law is established based on Chinese design code. First define the elastic damage variables as d_c and d_t, or using a more compact notation $d_{c/t}$, are expressed as follows:

$$d_{c/t} = \begin{cases} 1 - \dfrac{\rho_{c/t} n_{c/t}}{n_{c/t} - 1 + x_{c/t}} & \text{, when } x_{c/t} \le 1 \\ 1 - \dfrac{\rho_{c/t}}{\alpha_{c/t}(x_{c/t} - 1)^2 + x_{c/t}} & \text{, when } x_{c/t} > 1 \end{cases} \tag{6}$$

$$x_{c/t} = \frac{\varepsilon_{c/t}}{\varepsilon_{c/t,r}},\ \rho_{c/t} = \frac{f_{c/t,r}}{E_c \varepsilon_{c/t,r}},\ n_{c/t} = \frac{1}{1 - \rho_{c/t}} \tag{7}$$

where $\alpha_{c/t}$ = compressive/tensile descending parameter which controls the shape of post-peak part of stress-strain curve; $f_{c/t,r}$ = compressive/tensile peak strength; $\varepsilon_{c/t,r}$ = strain corresponding to the peak strength in compression/tension. The values of $f_{c/t,r}$, $\varepsilon_{c/t,r}$, $\alpha_{c/t}$ are recommended in Chinese design code, which are listed in Table 1 and Table 2.

Table 1 Parameters of compression (absolute value)

$f_{c,r}$ (*N/mm²*)	20	25	30	35	40	45	50	55	60	65	70	75	80
$\varepsilon_{c,r}$ (10^{-6})	1470	1560	1640	1720	1790	1850	1920	1980	2030	2080	2130	2190	2240
α_c	0.74	1.06	1.36	1.65	1.94	2.21	2.48	2.74	3.00	3.25	3.50	3.75	3.99

Table 2 Parameters of tension (absolute value)

$f_{t,r}$ (N/mm²)	1.0	1.5	2.0	2.5	3.0	3.5	4.0
$\varepsilon_{c,r}$ (10^{-6})	65	81	95	107	118	128	137
α_t	0.31	0.70	1.25	1.95	2.81	3.82	5.00

Hence the elasto-plastic compressive damage can be derived as:

$$\sigma = (1-d_c)E_c\varepsilon_c = (1-d_c^{'})E_c(\varepsilon_c - \varepsilon_p) \Rightarrow d_c^{'} = \frac{d_c - \phi_p}{1-\phi_p} \tag{8}$$

In the tensile part the plastic evolution is neglected and the damage variable is modified to account for the influence of the compressive damage:

$$d_{tc} = 1-(1-d_c)(1-d_t) \tag{9}$$

2. Command Manual

The model is implemented into OpenSees and the command is introduced now.

uniaxialMaterial ConcreteD $matTag $fc $epsc $ft $epst $Ec $alphac $alphat <$cesp> <$etap>

Parameters in the command:

$matTag	integer tag identifying material
$fc	concrete compressive strength
$epsc	concrete strain corresponding to compressive strength
$ft	concrete tensile strength
$epst	concrete strain corresponding to tensile strength
$Ec	concrete initial elastic modulus
$alphac	compressive descending parameter
$alphat	tensile descending parameter
$cesp	plastic parameter, recommended values: 0.2~0.3
$etap	plastic parameter, recommended values: 1.0~1.3

NOTE:

- Concrete compressive strength and the corresponding strain should be input as negative values.
- The value $fc/$epsc and $ft/$epst should be smaller than $Ec.
- The default value for $cesp and $etap are 0.25 and 1.15, respectively.

3. Numerical Examples

Example 1: Simulation of compressive test in Karson and Jirsa (1969).

uniaxialMaterial ConcreteD 1 -27.6 -0.002 3 0.0001 35000 1.0 0.1 0.25 1.15

 # the concrete material with tag 1 reaches compressive strength of 27 MPa at strain of 0.002 and reaches tensile strength of 3 MPa at strain of 0.0001, the initial elastic modulus is 35000 MPa, the descending parameter for compression is 1.0 while 0.1 for tensile, the plastic parameters are 0.25 and 1.15, respectively.

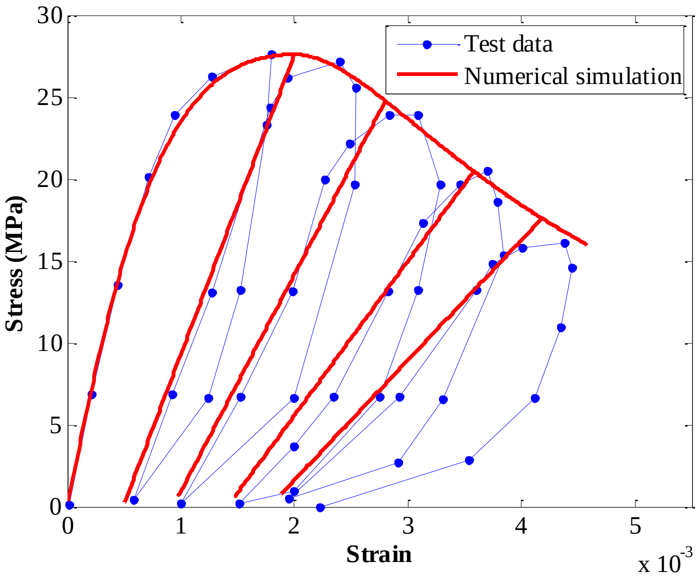
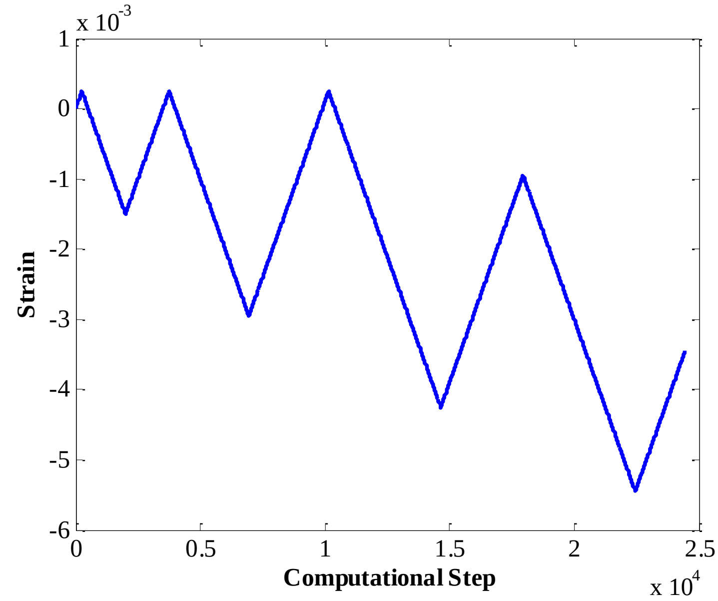


Fig. 2. Simulation of Karson & Jirsa test (1969)

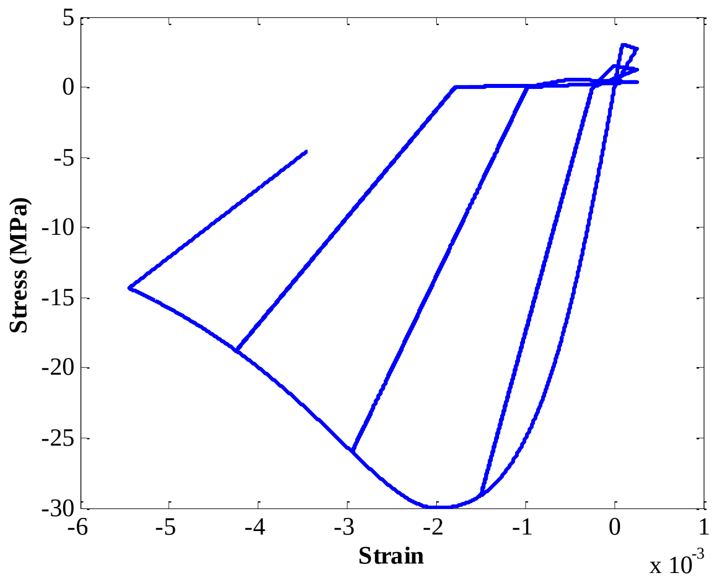
Example 2: Cyclic tension and compression response.

uniaxialMaterial ConcreteD 1 -30 -0.002 3 0.0001 35000 1.0 0.1 0.25 1.15

 # the concrete material with tag 1 reaches compressive strength of 30 MPa at strain of 0.002 and reaches tensile strength of 3 MPa at strain of 0.0001, the initial elastic modulus is 35000 MPa, the descending parameter for compression is 1.0 while 0.1 for tensile, the plastic parameters are 0.25 and 1.15, respectively.



(a) Stain history



(b) Cyclic stress-strain curve

Fig. 3. Cyclic tension and compression response

4. Reference

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[3] Ren, X. D. (2010). *Multi-scale based stochastic damage constitutive theory for concrete.*

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[5] Zeng, S. J. (2012). *Dynamic Experimental Research and Stochastic Damage Constitutive Model for Concrete*. Doctoral dissertation, Tongji University, Shanghai. (in Chinese)