混凝土结构基本原理

第五章　受弯构件正截面分析与计算

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➢受弯构件——梁

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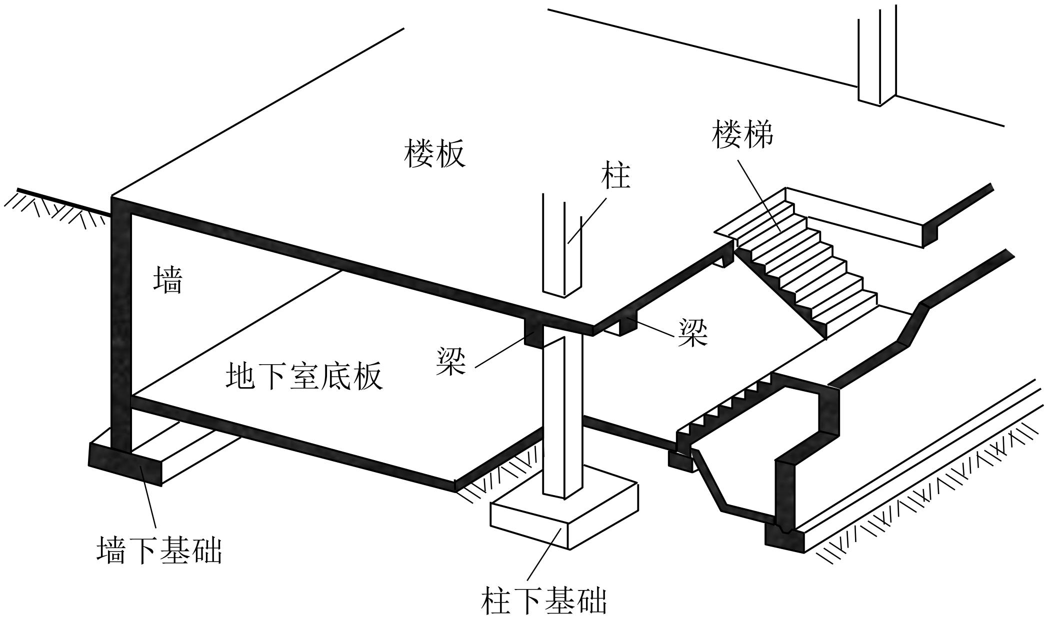
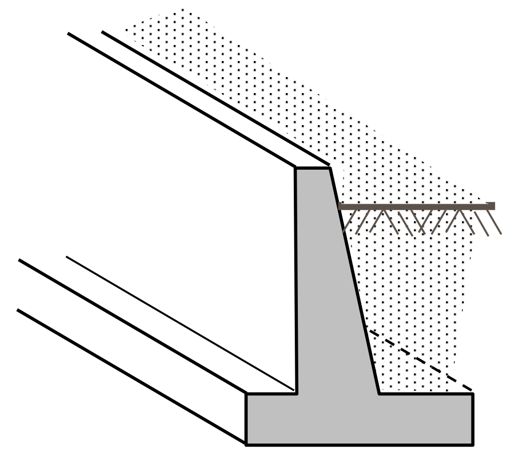
TONGJI UNIVERSITY

➢受弯构件工程实例

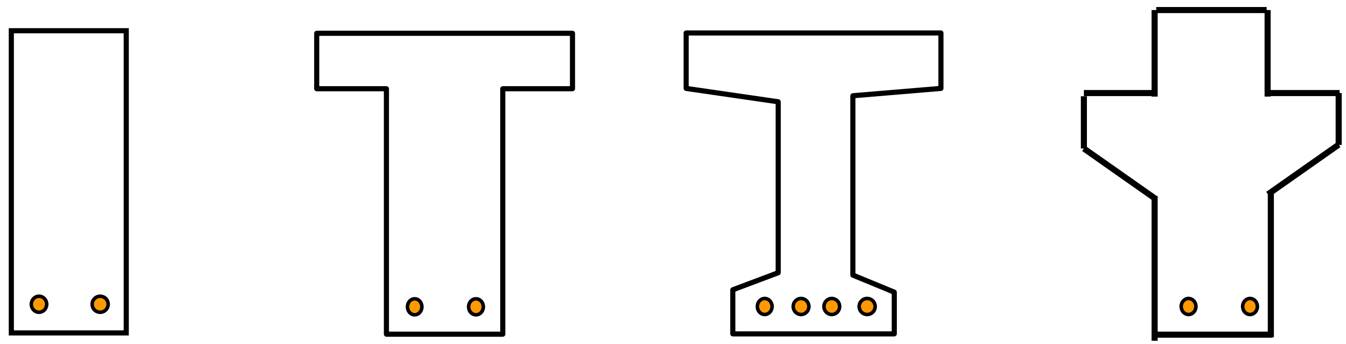




➢受弯构件工程实例

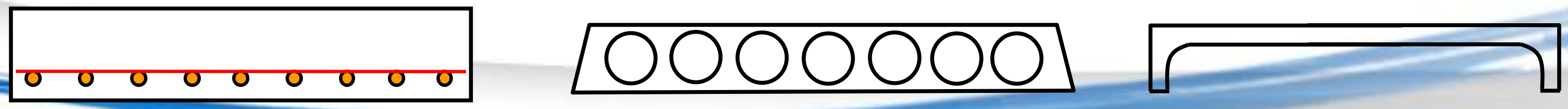
➢受弯构件主要截面形式

梁截面



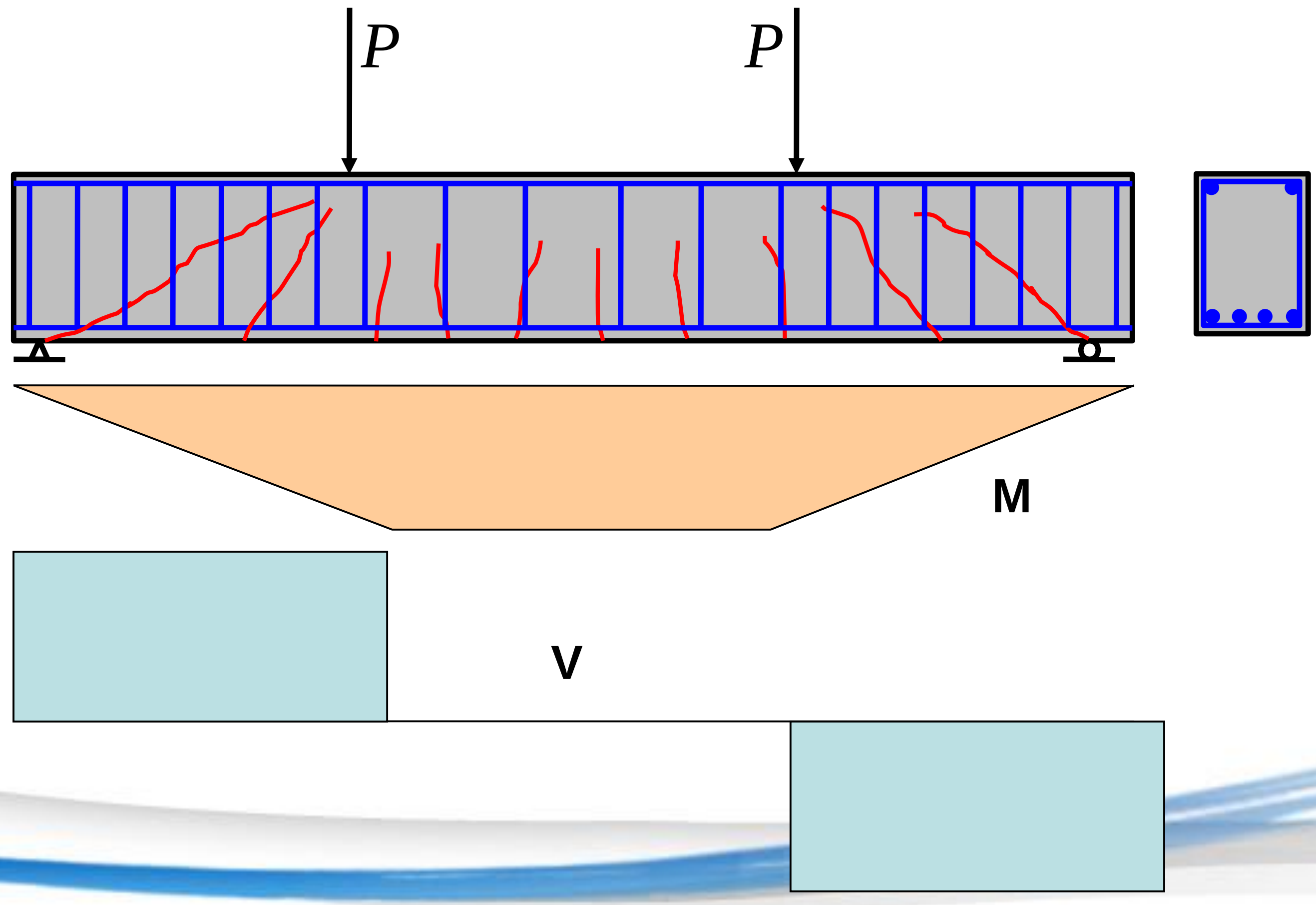
矩形梁　T形梁　I形梁　花篮梁

板截面

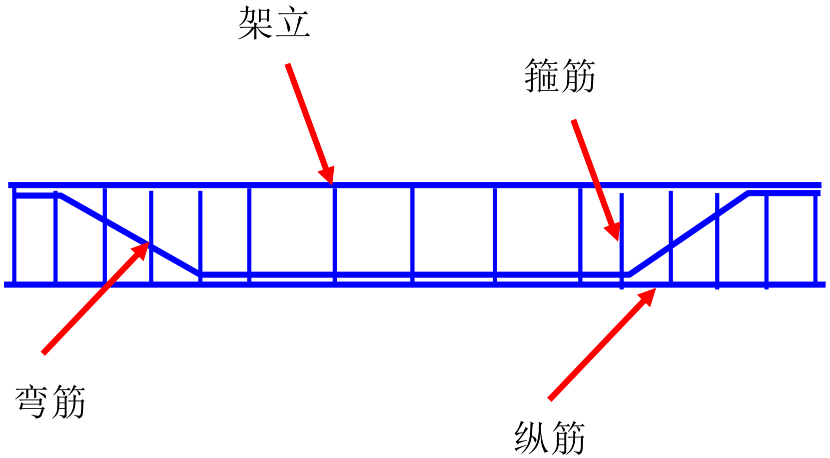


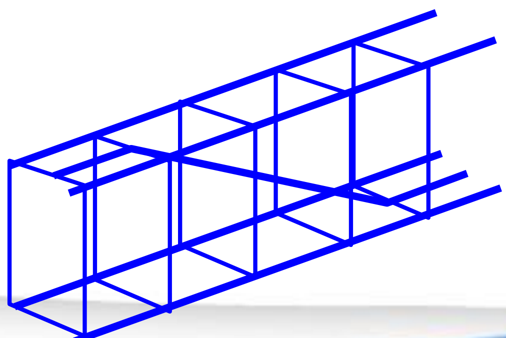
实心板　多孔板　槽形板

➢受弯构件的配筋形式

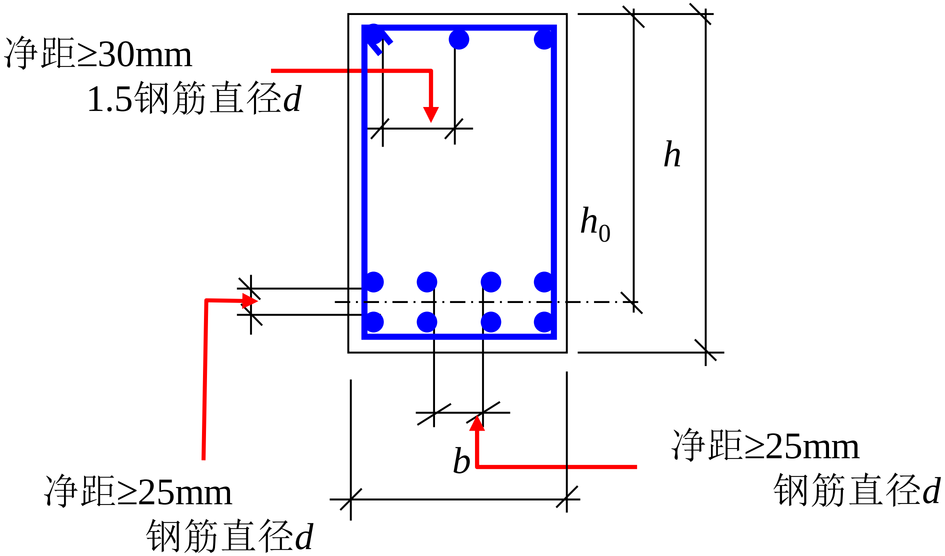


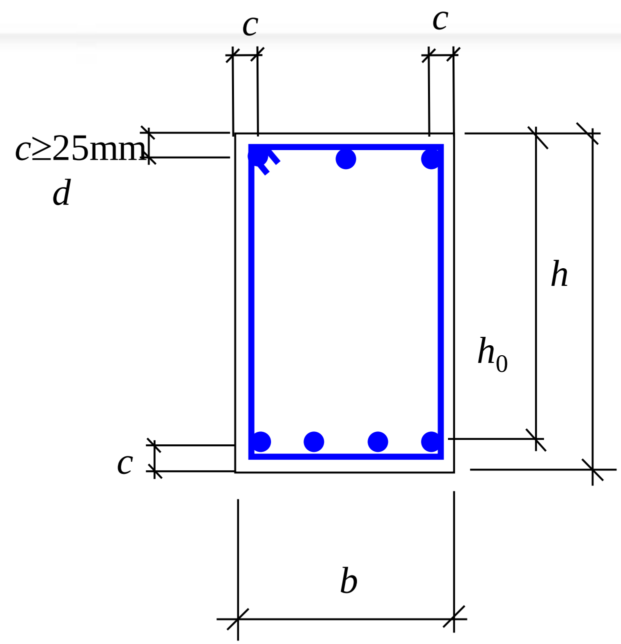
➢梁截面的配筋——弯起钢筋





➢梁的截面配筋构造

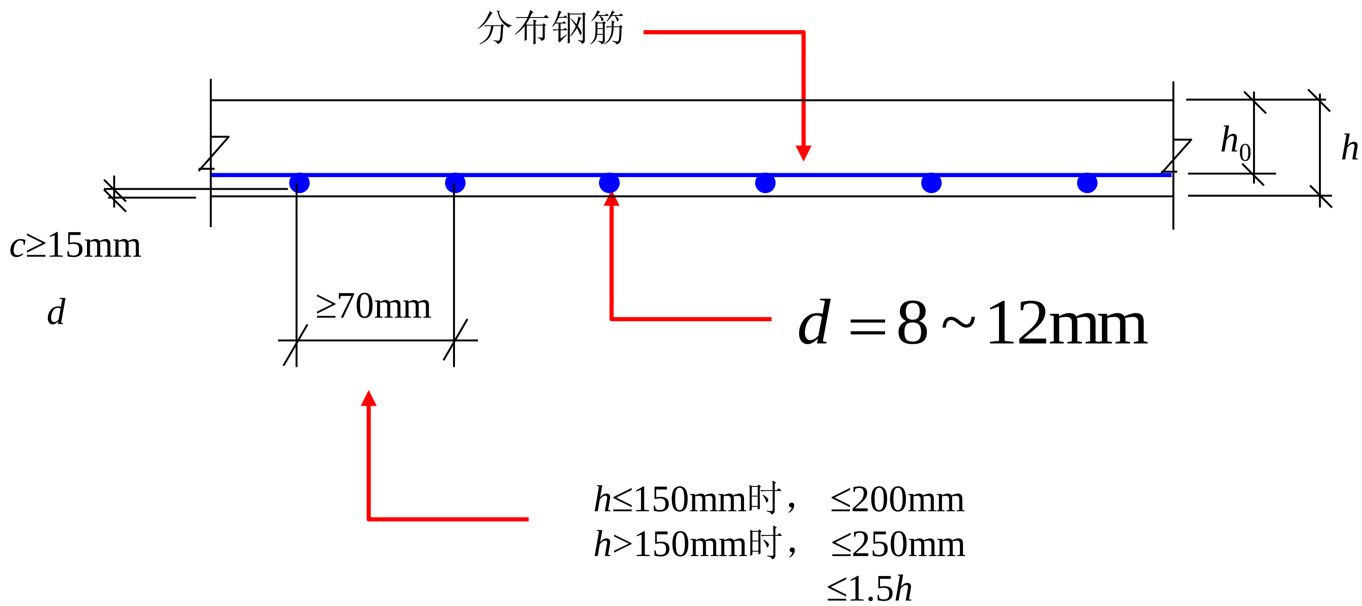




$$\frac{h}{b}=\begin{cases}2\sim3.5(\text{矩形截面})\\2.5\sim4.0(\text{T形截面})\end{cases}$$

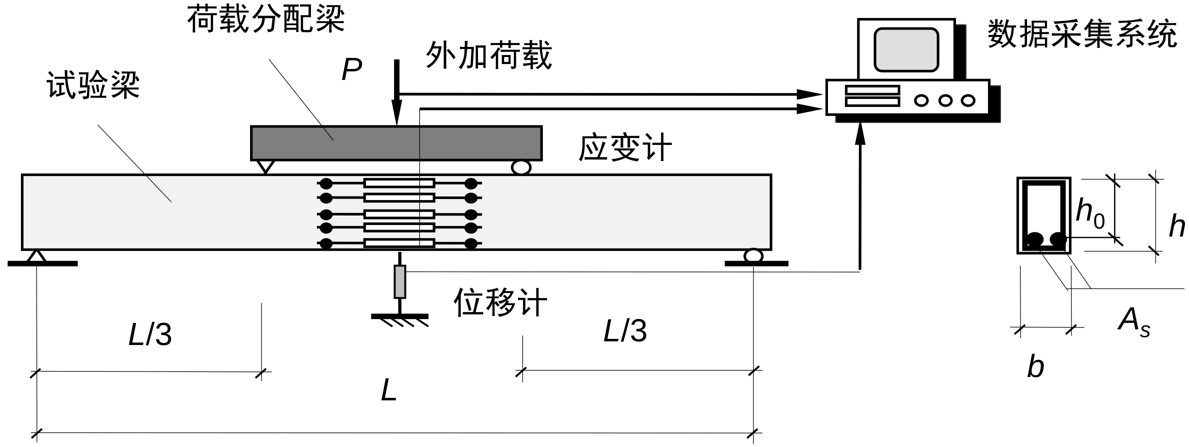
$$d=10\sim28\text{mm}(\text{桥梁中}14\sim40\text{mm})$$

➢板的截面配筋构造



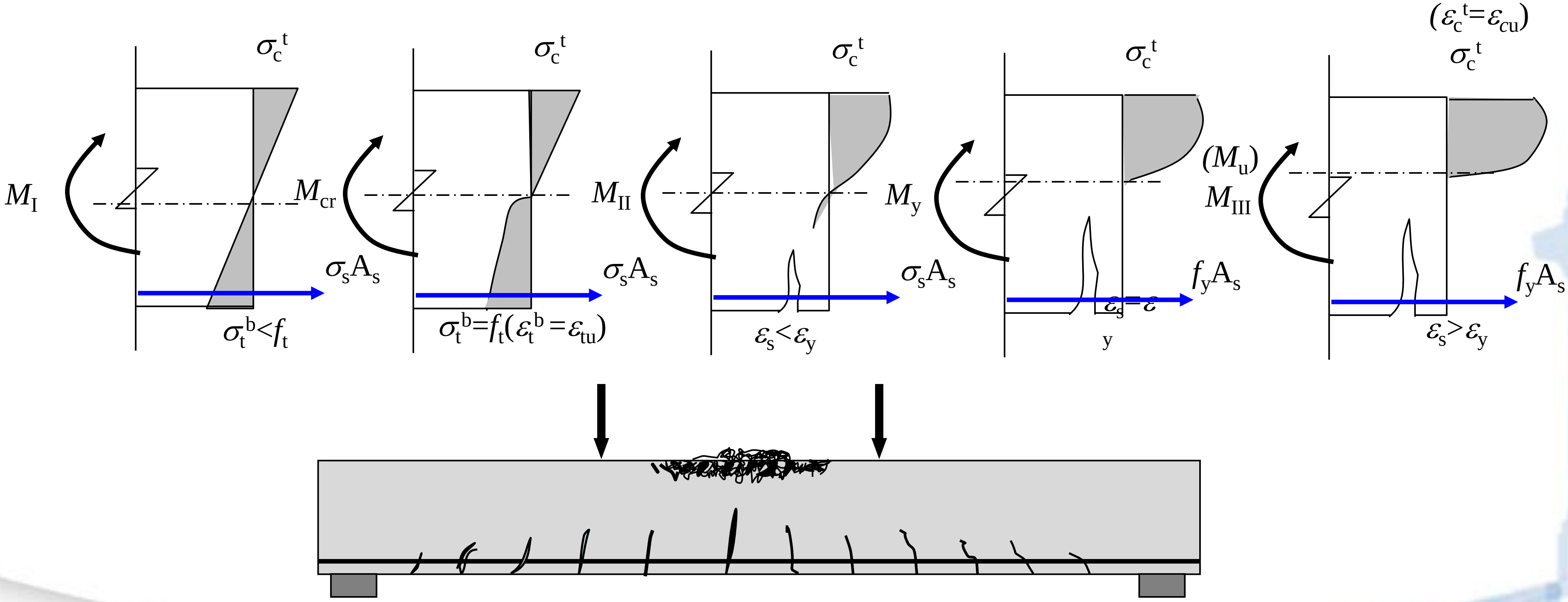
板厚的模数为10mm

➢受弯构件试验研究



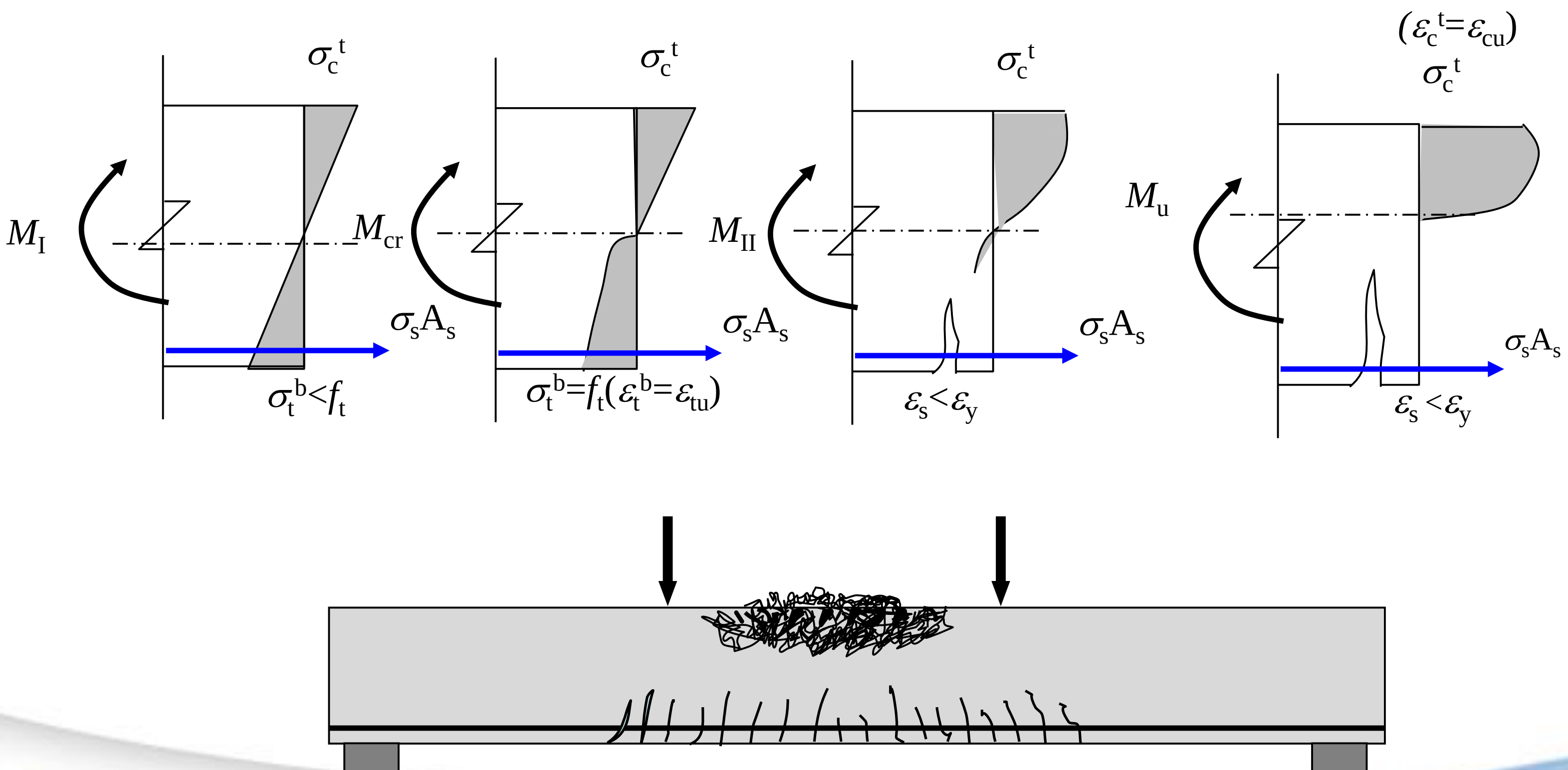
$$\rho = \frac{A_s}{bh_0}$$

➢试验结果——适筋破坏



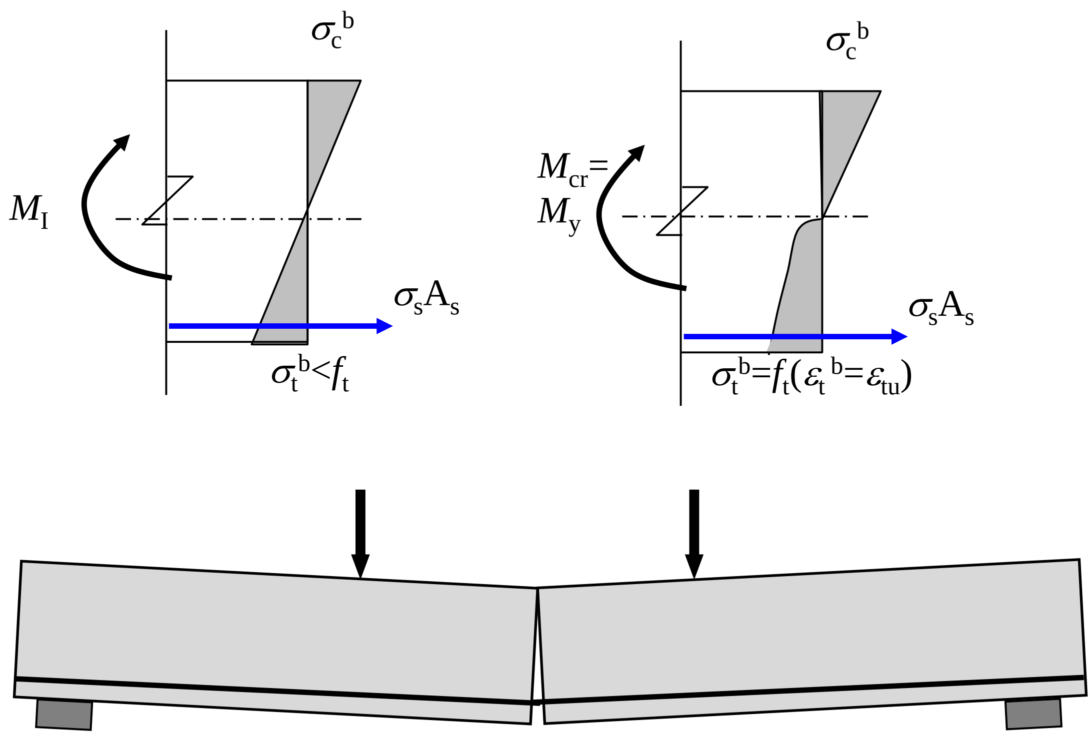
σ_c^t
M_I
$\sigma_s A_s$
$\sigma_t^b < f_t$
σ_c^t
M_{cr}
$\sigma_s A_s$
$\sigma_t^b = f_t(\varepsilon_t^b = \varepsilon_{tu})$
σ_c^t
M_{II}
$\sigma_s A_s$
$\varepsilon_s < \varepsilon_y$
σ_c^t
M_y
$f_y A_s$
$\varepsilon_s = \varepsilon_y$
$(\varepsilon_c^t = \varepsilon_{cu})$
σ_c^t
(M_u)
M_{III}
$f_y A_s$
$\varepsilon_s > \varepsilon_y$

➢试验结果——超筋破坏

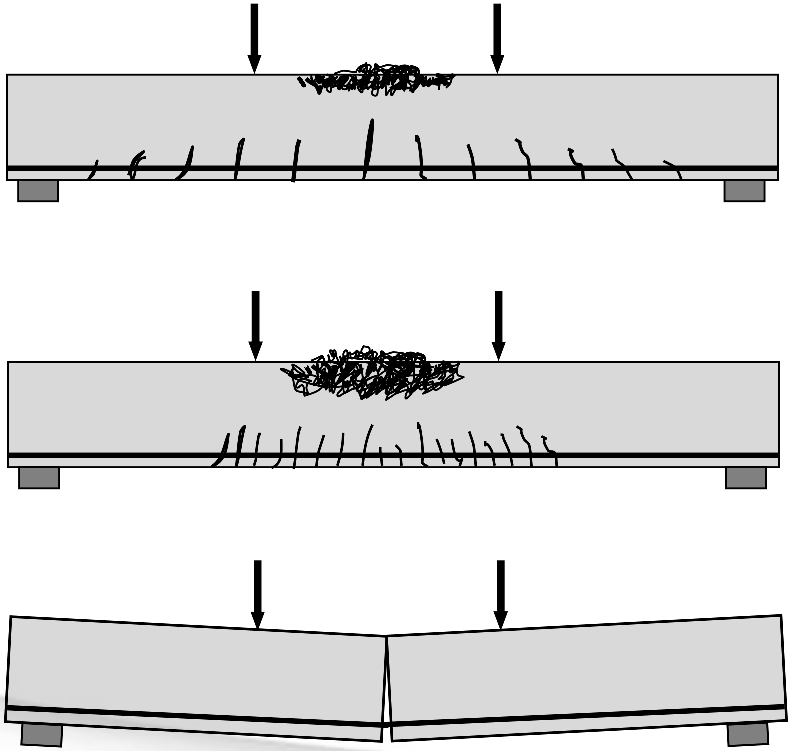


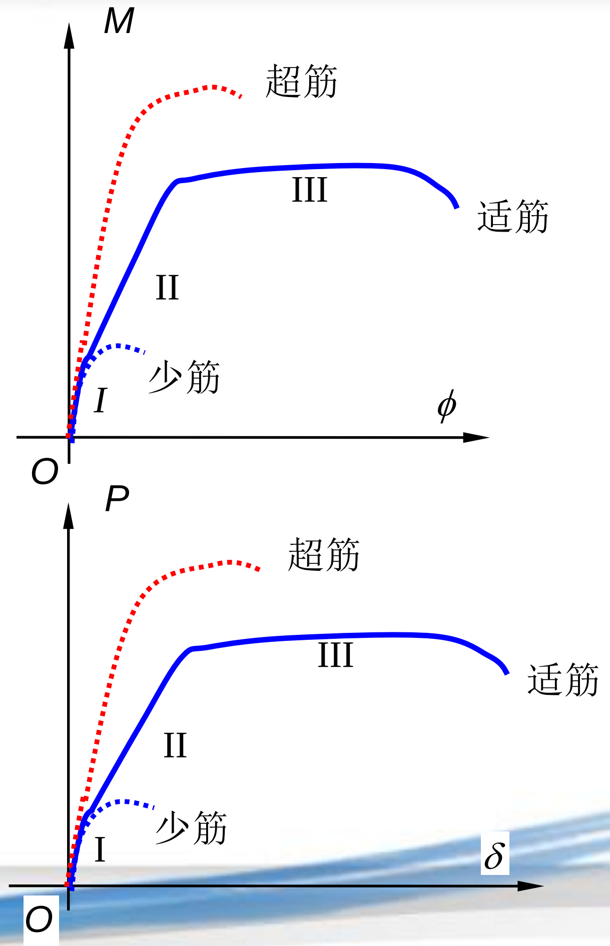
M_I
σ_c^t
$\sigma_s A_s$
$\sigma_t^b < f_t$
M_{cr}
σ_c^t
$\sigma_s A_s$
$\sigma_t^b = f_t(\varepsilon_t^b = \varepsilon_{tu})$
M_{II}
σ_c^t
$\sigma_s A_s$
$\varepsilon_s < \varepsilon_y$
M_u
$(\varepsilon_c^t = \varepsilon_{cu})$
σ_c^t
$\sigma_s A_s$
$\varepsilon_s < \varepsilon_y$

➢试验结果——少筋破坏


σ_c^b
M_I
$\sigma_s A_s$
$\sigma_t^b < f_t$
σ_c^b
$M_{cr}=$
M_y
$\sigma_s A_s$
$\sigma_t^b = f_t(\varepsilon_t^b = \varepsilon_{tu})$

➢试验结果

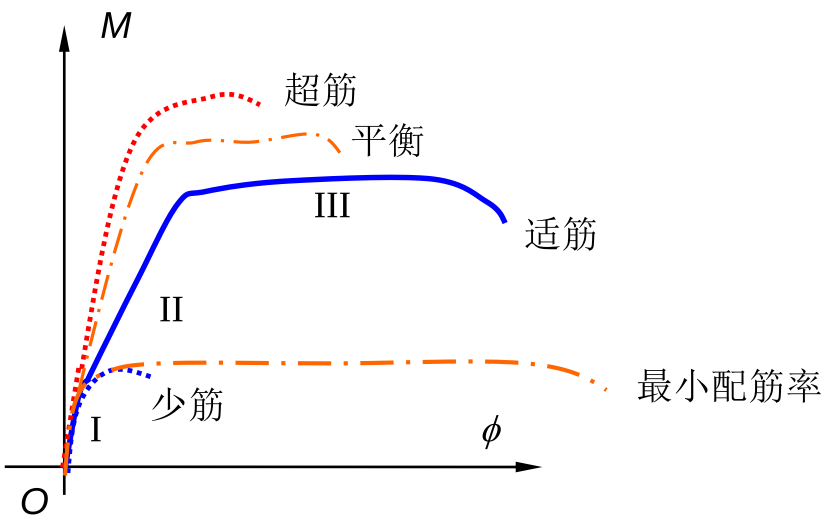


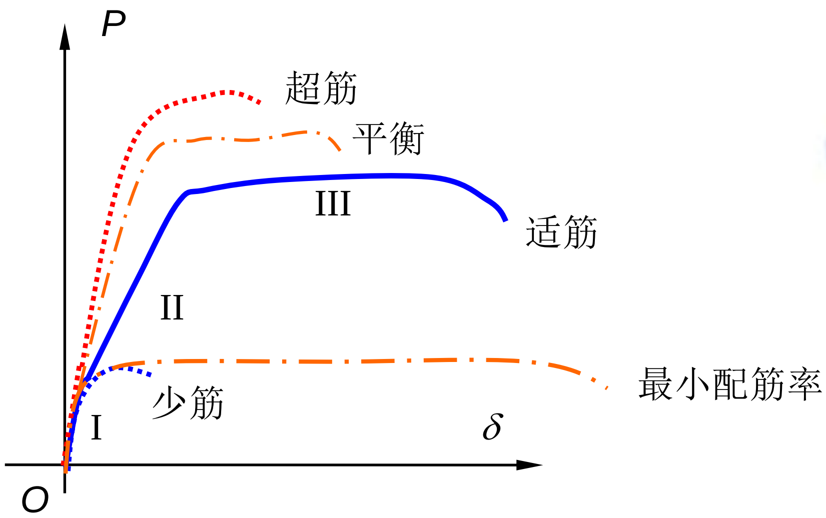


➢试验结论

- 适筋梁具有较好的变形能力，超筋梁和少筋梁的破坏具有突然性，设计时应予避免。
- 在适筋和超筋破坏之间存在一种平衡破坏。其破坏特征是钢筋屈服的同时，混凝土压碎，是区分适筋破坏和超筋破坏的定量指标。
- 在适筋和少筋破坏之间也存在一种“界限”破坏。其破坏特征是屈服弯矩和开裂弯矩相等，是区分适筋破坏和少筋破坏的定量指标。

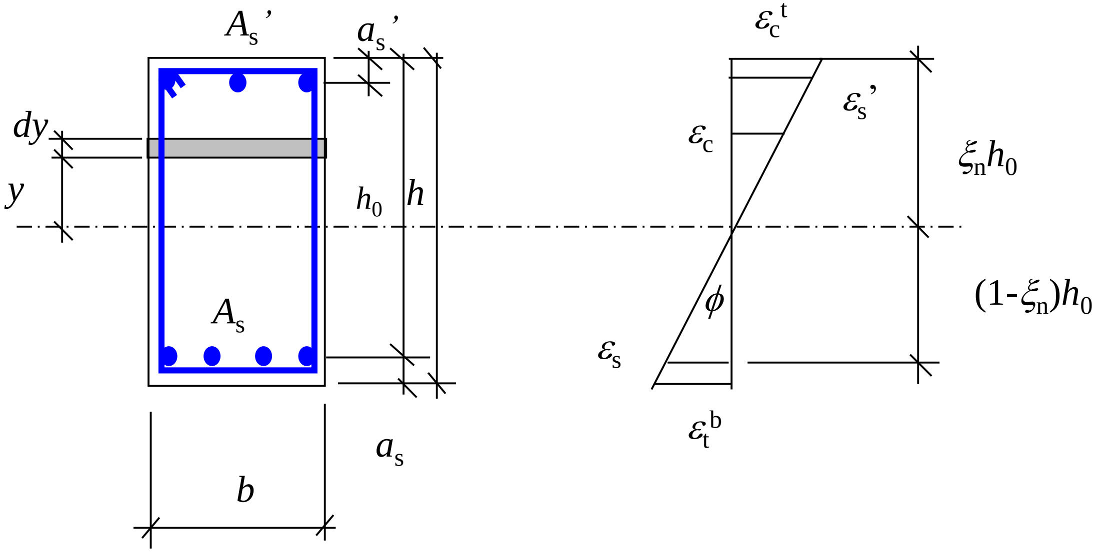
➢荷载—位移曲线





➤受弯构件受力分析

- 平截面假定

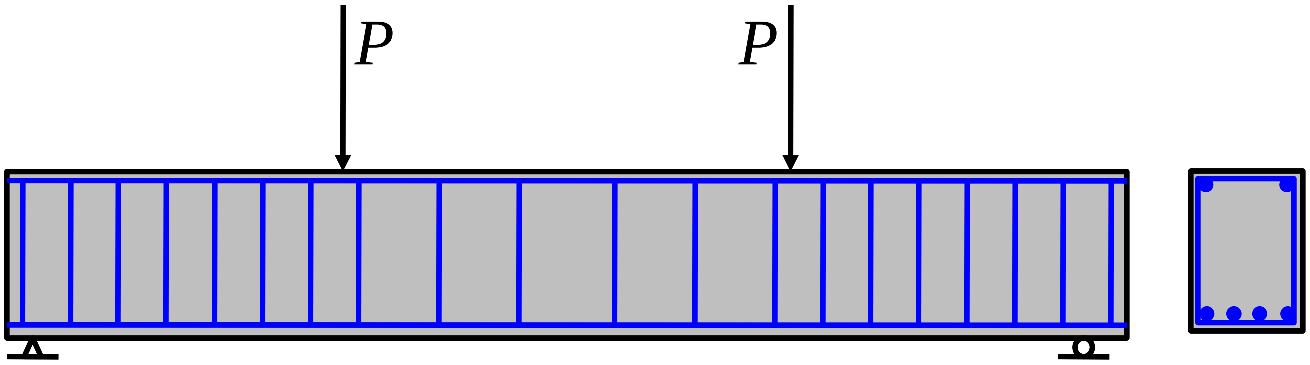


开裂后：
平均应变平截面假定

$$\varphi=\frac{\varepsilon_c^t}{\xi_n h_0}=\frac{\varepsilon_c}{y}=\frac{\varepsilon_s'}{\xi_n h_0-a_s'}=\frac{\varepsilon_s}{(1-\xi_n)h_0}$$

➢受弯构件受力分析

- 钢筋的应变和相同位置处混凝土的应变相同
  - ——假定混凝土与钢筋之间粘结可靠



➢受弯构件受力分析

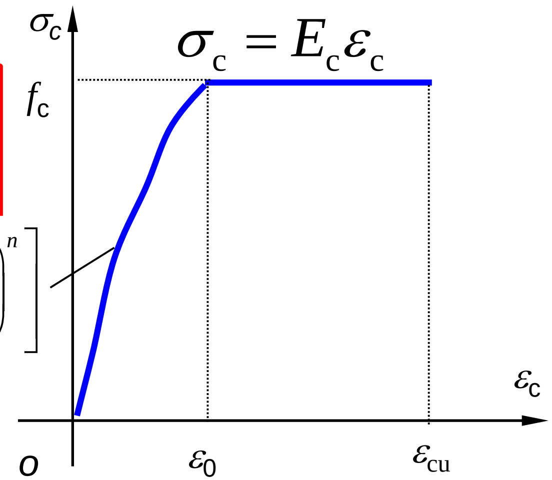
- 混凝土受压本构关系

当应力较小时，如$\sigma_c < 0.3 f_c$时，可取

$$\sigma_c = E_c \varepsilon_c$$

$$n = 2 - \frac{1}{60}(f_{cu} - 50)$$，当$n \geq 2$时，取$n = 2$

$$\sigma_c = f_c\left[1 - \left(1 - \frac{\varepsilon_c}{\varepsilon_0}\right)^n\right]$$



$$\varepsilon_0 = 0.002 + 0.5(f_{cu} - 50) \times 10^{-5}$$

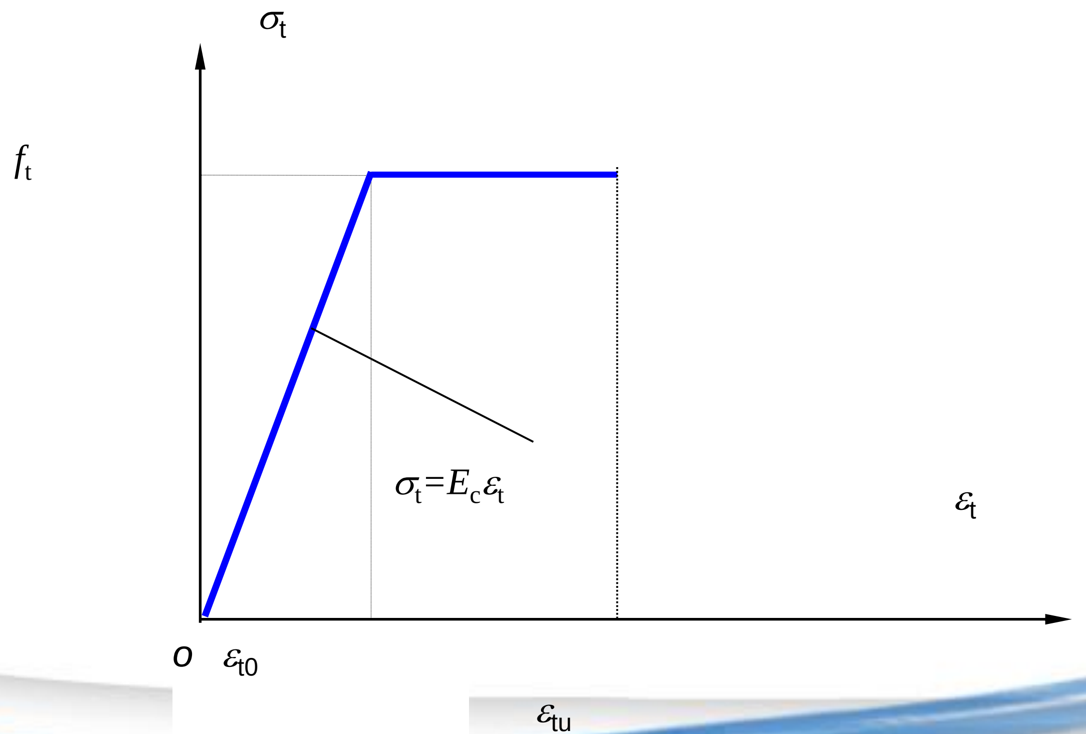
$\varepsilon_0 < 0.002$时，取$\varepsilon_0 = 0.002$

$$\varepsilon_{cu} = 0.0033 - (f_{cu} - 50) \times 10^{-5}$$

$\varepsilon_{cu} > 0.0033$时，取$\varepsilon_{cu} = 0.0033$

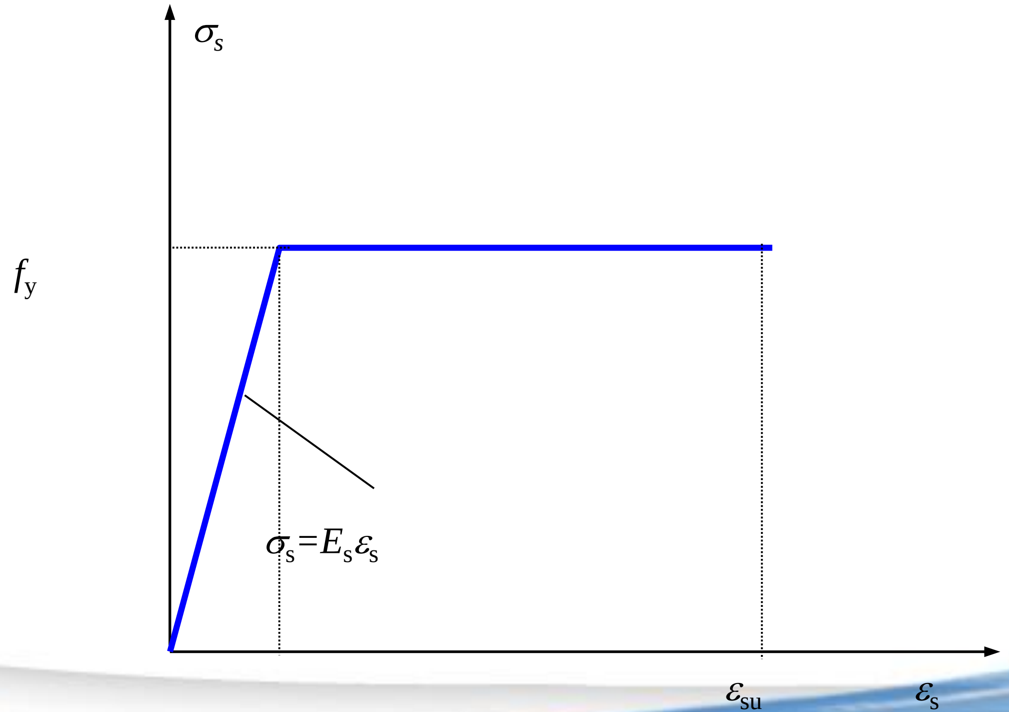
➢受弯构件受力分析

- 混凝土受拉本构关系



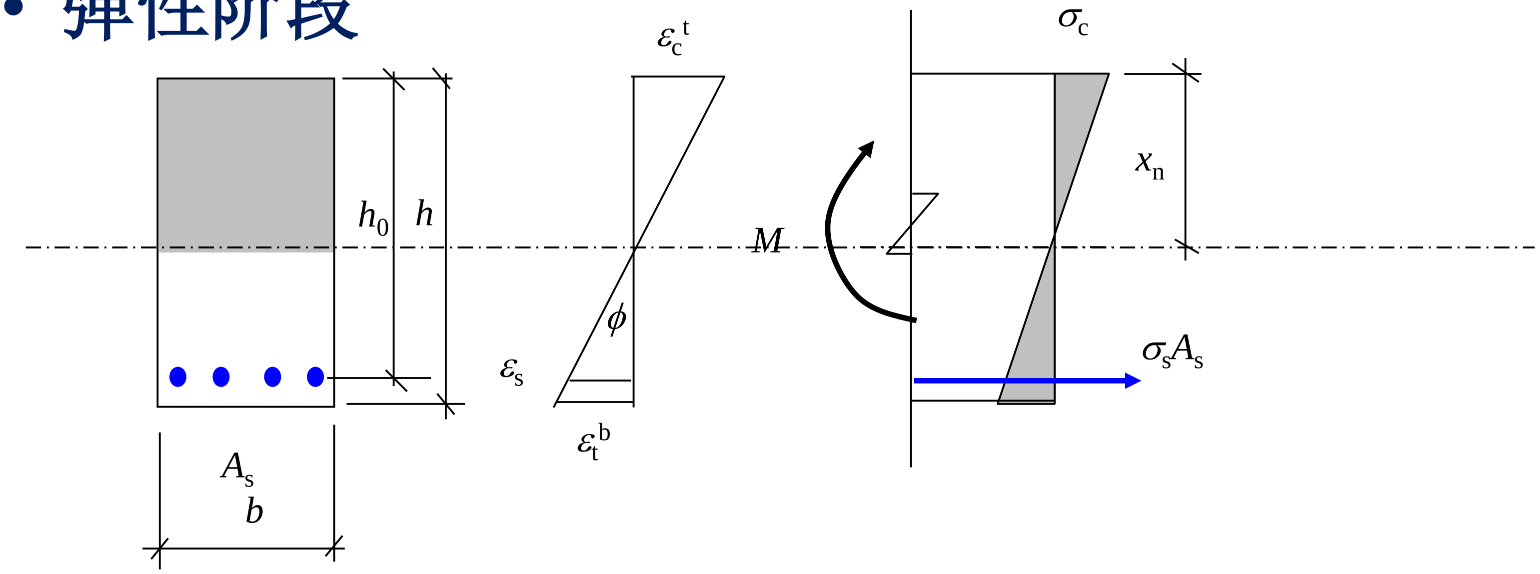
➢受弯构件受力分析

- 钢筋本构关系



➢受弯构件受力分析

- 弹性阶段

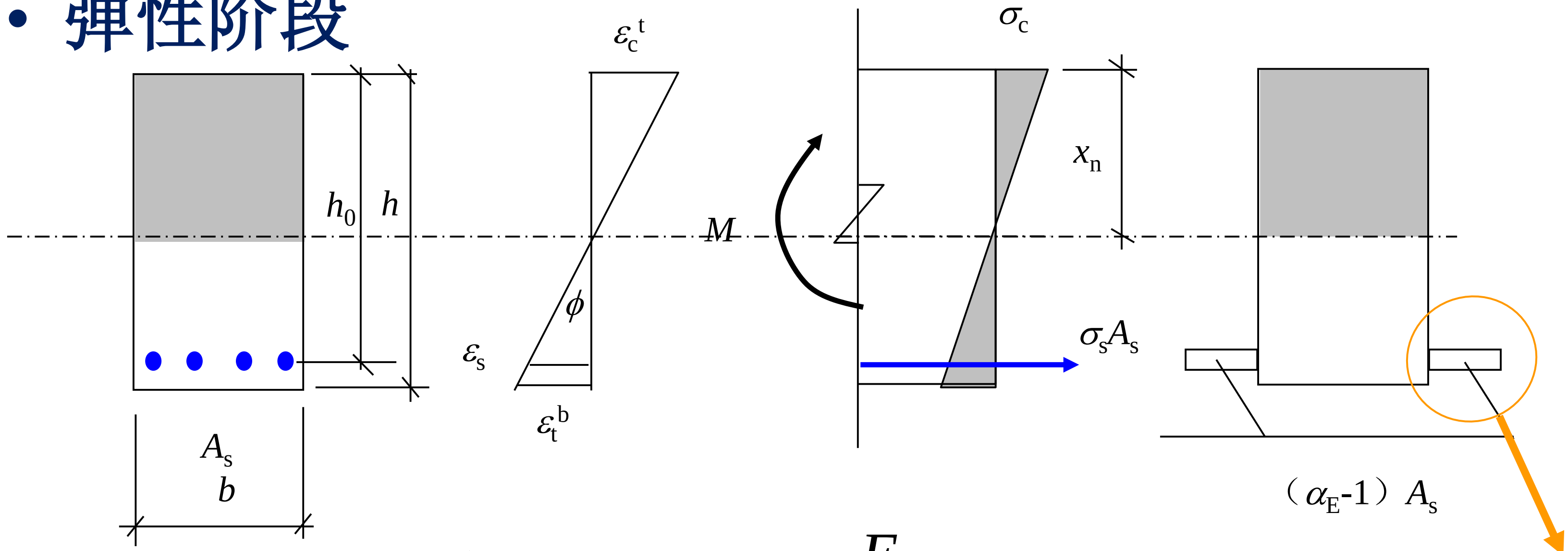


采用线形的物理关系

$$\varepsilon_c = \sigma_c / E_c \qquad \varepsilon_t = \sigma_t / E_c \qquad \varepsilon_s = \sigma_s / E_s$$

➢受弯构件受力分析

- 弹性阶段



$$\varepsilon_s=\varepsilon_t \quad\Rightarrow\quad \sigma_s=E_s\varepsilon_s=\frac{E_s}{E_c}\sigma_t=\alpha_E\sigma_t$$

用材料力学的方法求解

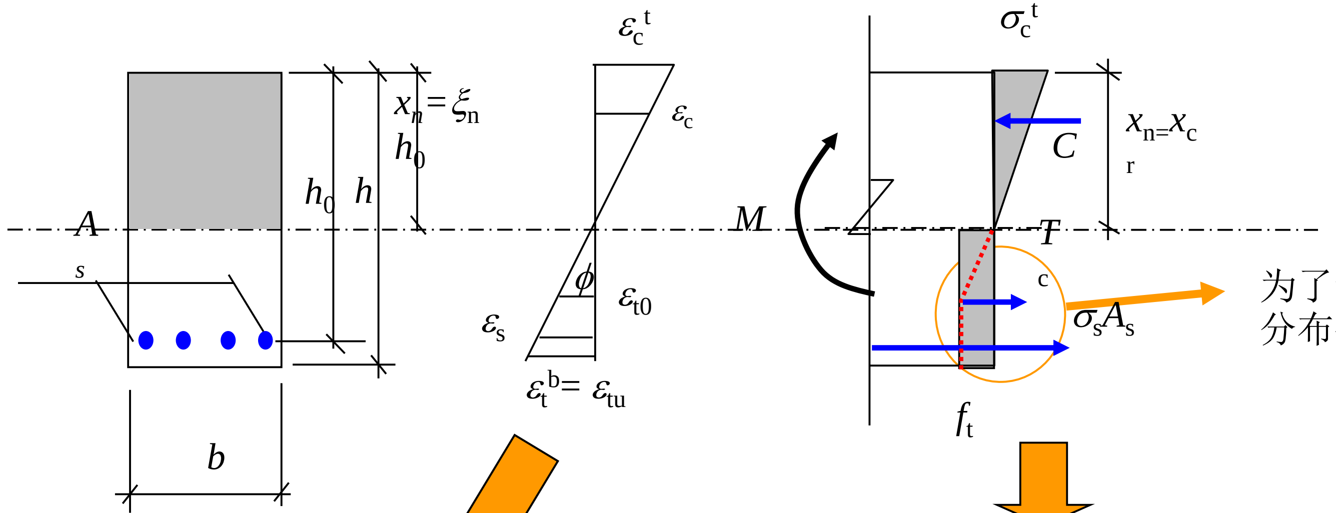
$$T=\sigma_s A_s=\alpha_E A_s\sigma_t$$

将钢筋等效成混凝土

➢受弯构件受力分析

- **开裂点**

当$\varepsilon_t^b=\varepsilon_{tu}$时认为拉区混凝土开裂并退出工作（约束受拉）

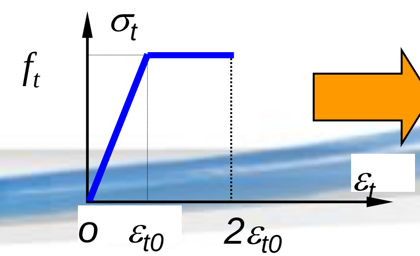


为了计算方便用矩形应力分布代替原来的应力分布

$$\varphi=\frac{\varepsilon_{tu}}{h-x_{cr}}=\frac{\varepsilon_c^t}{x_{cr}}=\frac{\varepsilon_s}{h_0-x_{cr}}$$

$$\sigma_c^t=E_c\varepsilon_c$$

$$\sigma_s=E_s\varepsilon_s$$



$$f_t=0.5E_c\varepsilon_{tu}$$

➢受弯构件受力分析

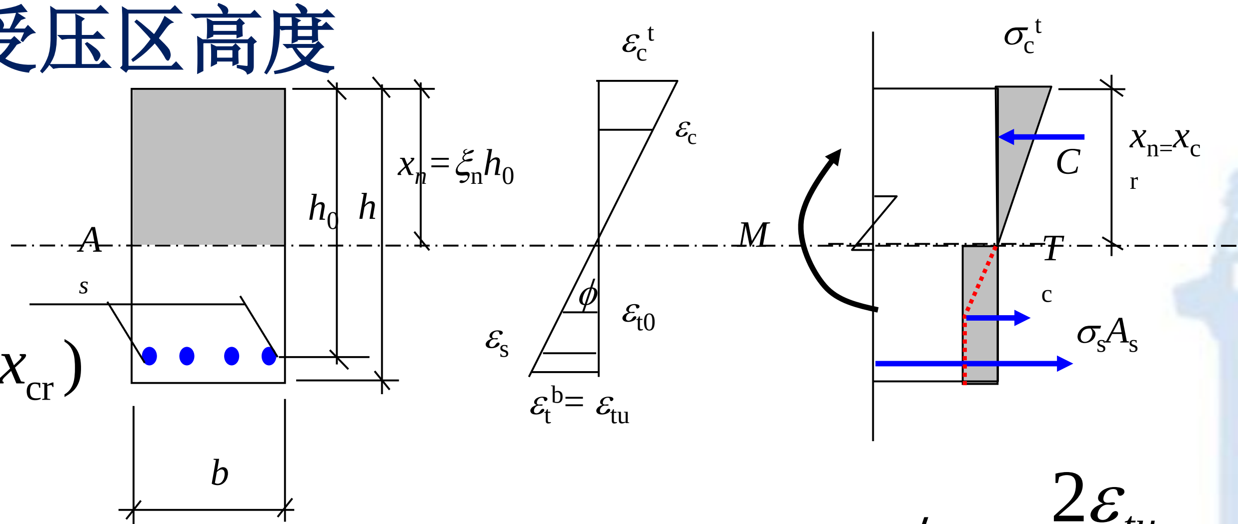
- 开裂点截面受压区高度

$$\sum X = 0$$

$$0.5\sigma_c^t b x_{cr} = 0.5E_c\varepsilon_{tu} b(h - x_{cr}) + \sigma_s A_s$$

设 $\alpha_E = \dfrac{E_s}{E_c}$，近似认为 $\varepsilon_s = \varepsilon_{tu}$

$$x_{cr} = \frac{1+\dfrac{2\alpha_E A_s}{bh}}{1+\dfrac{\alpha_E A_s}{bh}} \times \frac{h}{2}$$



对一般钢筋混凝土梁

$A_s / bh = 0.5 \sim 2\%$,

$\alpha_E = 6 \sim 7$

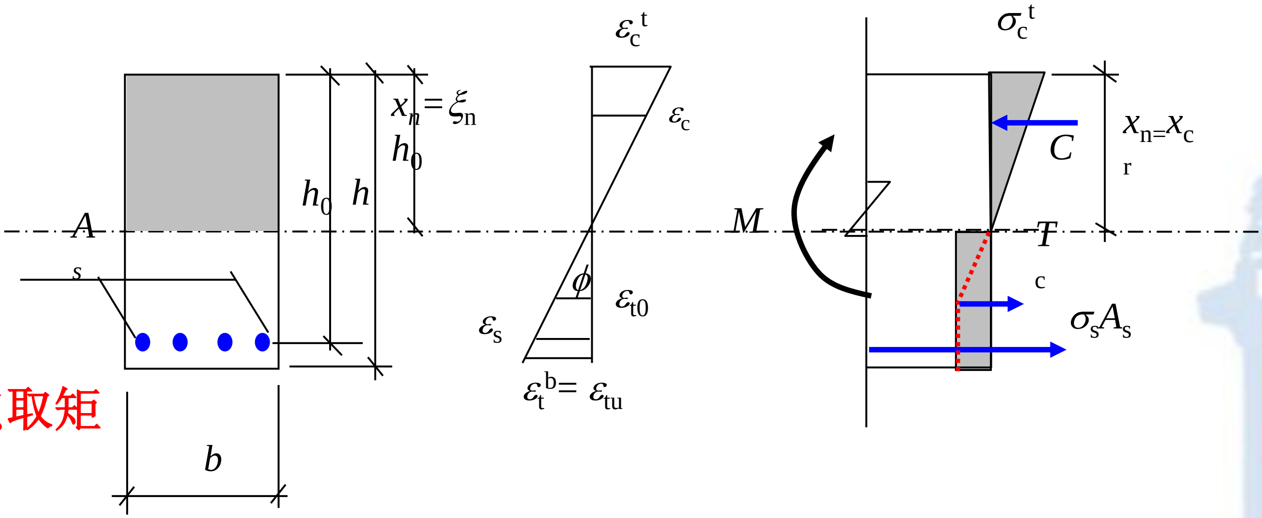
$$x_{cr} \approx 0.5h$$

$$\phi_{cr} = \frac{2\varepsilon_{tu}}{h}$$

➢受弯构件受力分析

- 开裂点弯矩

$$\sum M = 0$$



向受压合力点取矩

$$M_{cr} = f_t b(h-x_{cr})\left(\frac{h-x_{cr}}{2}+\frac{2x_{cr}}{3}\right)$$

$$+2\alpha_E f_t A_s\left(h_0-\frac{x_{cr}}{3}\right)$$

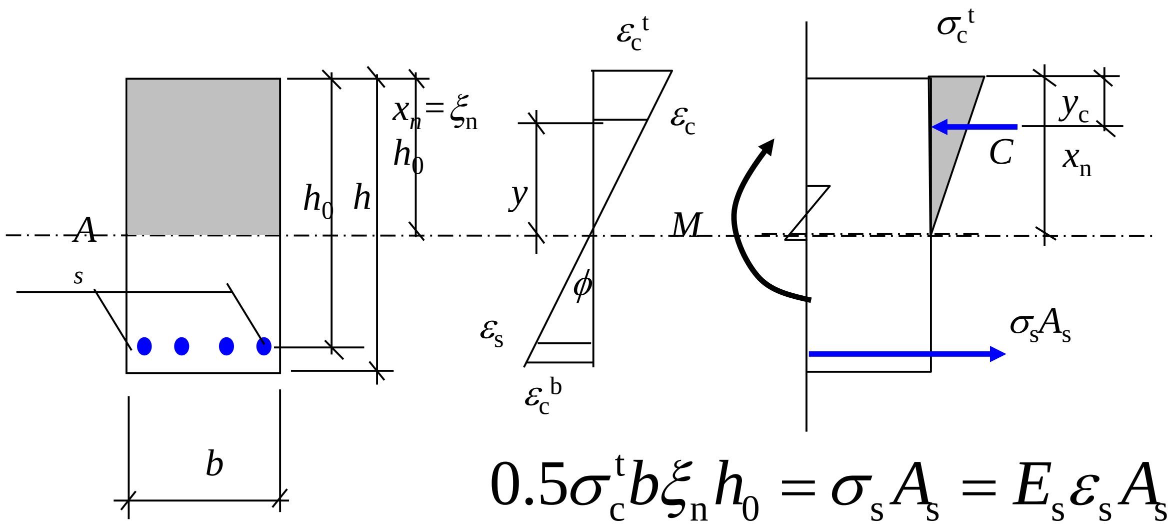
设$h_0 = 0.92h$, 令$\alpha_A = 2\alpha_E \dfrac{A_s}{bh}$

$$M_{cr} = 0.292(1+2.5\alpha_A) f_t bh^2$$

➢受弯构件受力分析

• 开裂前期——压区混凝土处于弹性状态

压区混凝土处于弹性阶段



M较小时，σ_c可以认为是按线性分布，忽略拉区混凝土的作用

$$0.5\sigma_c^t b\xi_n h_0 = \sigma_s A_s = E_s\varepsilon_s A_s = E_s \frac{(1-\xi_n)h_0}{\xi_n h_0}\varepsilon_c^t A_s$$

$$\sum X = 0$$

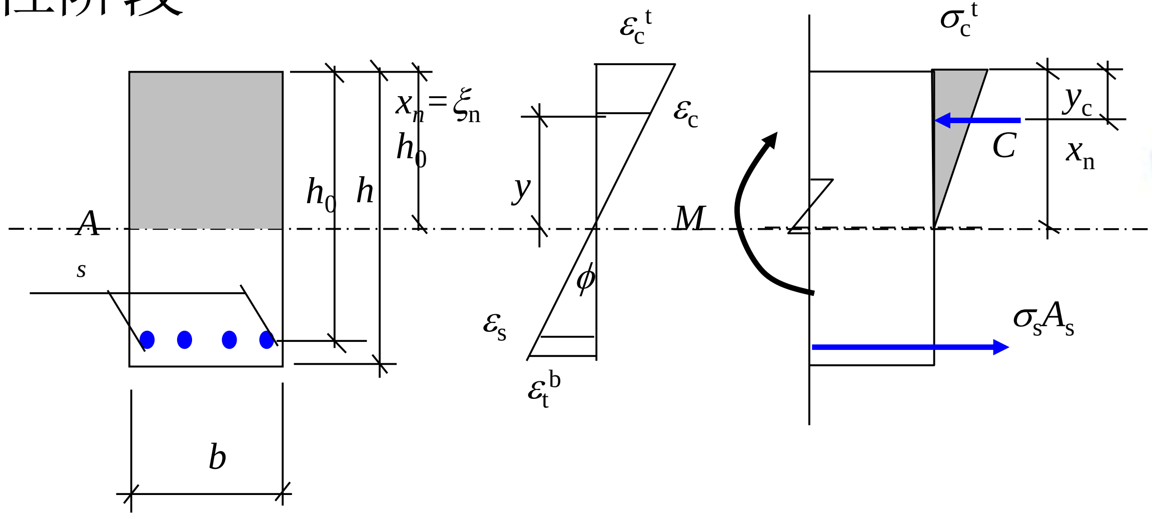
$$=\alpha_E \frac{1-\xi_n}{\xi_n}\sigma_c^t A_s$$

$$\xi_n^2 + 2\alpha_E\rho\xi_n - 2\alpha_E\rho = 0$$

➢受弯构件受力分析

- 开裂前期——压区混凝土处于弹性状态

压区混凝土处于弹性阶段



$$\sum M = 0$$

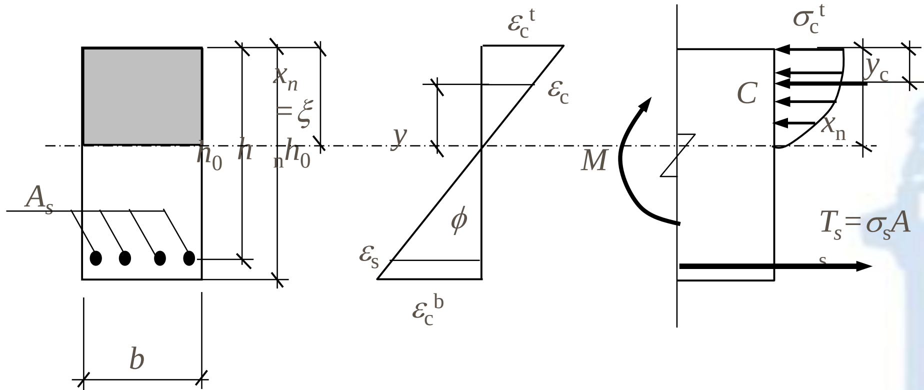
$$M = 0.5\sigma_c^t b\xi_n h_0^2\left(1-\frac{1}{3}\xi_n\right) = \sigma_s A_s h_0\left(1-\frac{1}{3}\xi_n\right)$$

➢受弯构件受力分析

• 开裂中期

压区混凝土处于弹塑性阶段，但$\varepsilon_c^t<\varepsilon_0$

(以混凝土强度等级不大于C50的钢筋混凝土受弯构件为例)



$$C = f_c b\int_0^{\xi_n h_0}\left(2\frac{\varepsilon_c}{\varepsilon_0}-\frac{\varepsilon_c^2}{\varepsilon_0^2}\right)dy = f_c b\int_0^{\xi_n h_0}\left(2\frac{\varepsilon_c^t}{\xi_n h_0\varepsilon_0}y-\frac{\varepsilon_c^{t\,2}}{\xi_n h_0\varepsilon_0^2}y^2\right)dy$$

$$= f_c b\xi_n h_0\left(\frac{\varepsilon_c^t}{\varepsilon_0}-\frac{\varepsilon_c^{t\,2}}{3\varepsilon_0^2}\right)$$

$$f_c b\xi_n h_0\left(\frac{\varepsilon_c^t}{\varepsilon_0}-\frac{\varepsilon_c^{t\,2}}{3\varepsilon_0^2}\right) = E_s\frac{1-\xi_n}{\xi_n}\varepsilon_c^t A_s$$

$$y_c = \xi_n h_0 - \frac{f_c b\int_0^{\xi_n h_0}\left(2\frac{\varepsilon_c}{\varepsilon_0}-\frac{\varepsilon_c^2}{\varepsilon_0^2}\right)y\,dy}{f_c b\int_0^{\xi_n h_0}\left(2\frac{\varepsilon_c}{\varepsilon_0}-\frac{\varepsilon_c^2}{\varepsilon_0^2}\right)dy} = \xi_n h_0\frac{\frac{1}{3}-\frac{\varepsilon_c^t}{12\varepsilon_0}}{1-\frac{\varepsilon_c^t}{3\varepsilon_0}}$$

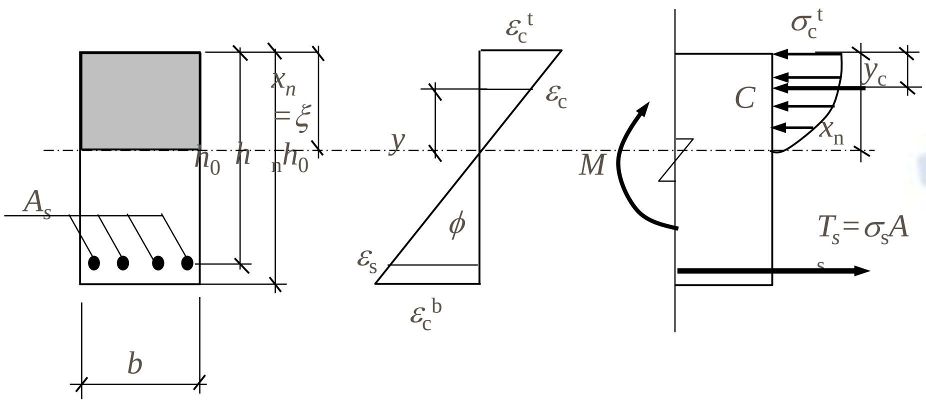
$$f_c\xi_n^2\left(\frac{1}{\varepsilon_0}-\frac{\varepsilon_c^t}{2\varepsilon_0^2}\right) = E_s(1-\xi_n)\rho$$

➢受弯构件受力分析

• 开裂中期

压区混凝土处于弹塑性阶段，但$\varepsilon_c^t<\varepsilon_0$

(以混凝土强度等级不大于C50的钢筋混凝土受弯构件为例)



$$M = f_c b \xi_n h_0^2 \left(\frac{\varepsilon_c^t}{\varepsilon_0} - \frac{\varepsilon_c^{t\,2}}{3\varepsilon_0^2} \right) \left(1 - \xi_n \frac{\dfrac{1}{3} - \dfrac{\varepsilon_c^t}{12\varepsilon_0}}{1 - \dfrac{\varepsilon_c^t}{3\varepsilon_0}} \right)$$

$$= \sigma_s A_s h_0 \left(1 - \xi_n \frac{\dfrac{1}{3} - \dfrac{\varepsilon_c^t}{12\varepsilon_0^2}}{1 - \dfrac{\varepsilon_c^t}{3\varepsilon_0}} \right) \qquad (\sigma_s \le f_y)$$

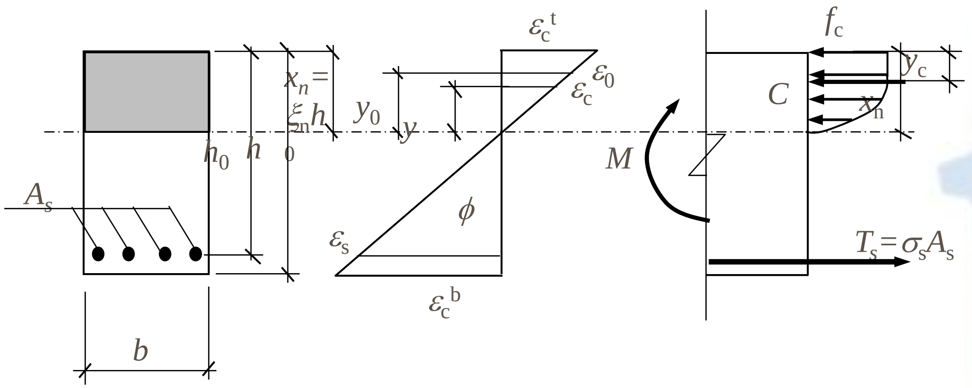
➢受弯构件受力分析

- 开裂后期

压区混凝土处于弹塑性阶段，但$\varepsilon_0 < \varepsilon_c^t < \varepsilon_{cu}$

(以混凝土强度等级不大于C50的钢筋混凝土受弯构件为例)

$$C = f_c b \xi_n h_0 \left(1 - \frac{1}{3}\frac{\varepsilon_0}{\varepsilon_c^t}\right)$$



$$f_c b \xi_n h_0 \left(1 - \frac{\varepsilon_0}{3\varepsilon_c^t}\right) = E_s \frac{1-\xi_n}{\xi_n} \varepsilon_c^t A_s$$

$$y_c = \xi_n h_0 \left(1 - \frac{\frac{1}{2} - \frac{1}{12}\left(\frac{\varepsilon_0}{\varepsilon_c^t}\right)^2}{1 - \frac{1}{3}\frac{\varepsilon_0}{\varepsilon_c^t}}\right)$$

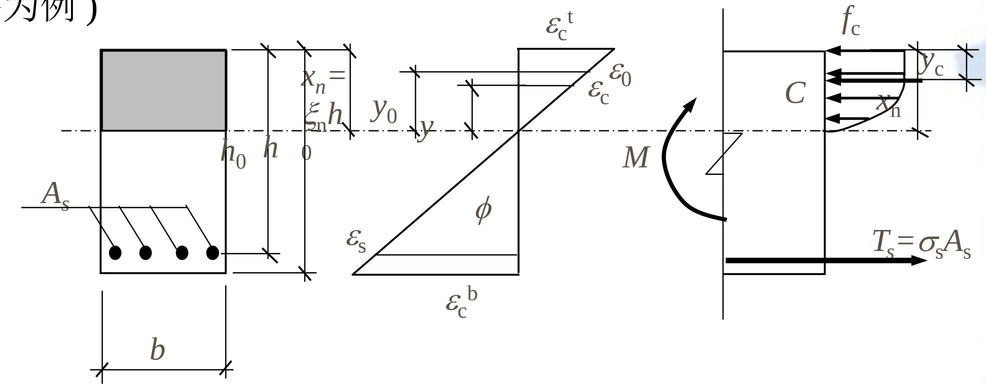
$$f_c \xi_n^2 \left(1 - \frac{\varepsilon_0}{3\varepsilon_c^t}\right) = E_s (1-\xi_n) \varepsilon_c^t \rho$$

➢受弯构件受力分析

• 开裂后期

压区混凝土处于弹塑性阶段，但$\varepsilon_0 < \varepsilon_c^t < \varepsilon_{cu}$

(以混凝土强度等级不大于C50的钢筋混凝土受弯构件为例)



$$M = f_c b \xi_n h_0 \left(1-\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_c^t}\right)\left\{1-\xi_n\left[1-\frac{\frac{1}{2}-\frac{1}{12}\left(\frac{\varepsilon_0}{\varepsilon_c^t}\right)^2}{1-\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_c^t}}\right]\right\}$$

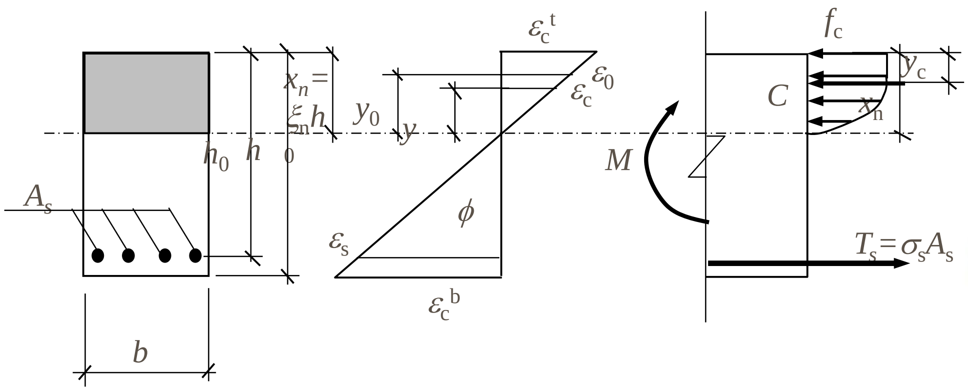
$$= \sigma_s A_s h_0\left\{1-\xi_n\left[1-\frac{\frac{1}{2}-\frac{1}{12}\left(\frac{\varepsilon_0}{\varepsilon_c^t}\right)^2}{1-\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_c^t}}\right]\right\} \qquad (\sigma_s \le f_y)$$

➢受弯构件受力分析

- 破坏阶段

$\varepsilon_c^t = \varepsilon_{cu}$。当$f_{cu} \le 50$Mpa时，

$n = 2, \varepsilon_0 = 0.002, \varepsilon_{cu} = 0.0033$



应用前面公式

$$f_c \xi_n^2 + 0.000055 E_s (1-\xi_n)\rho = 0$$

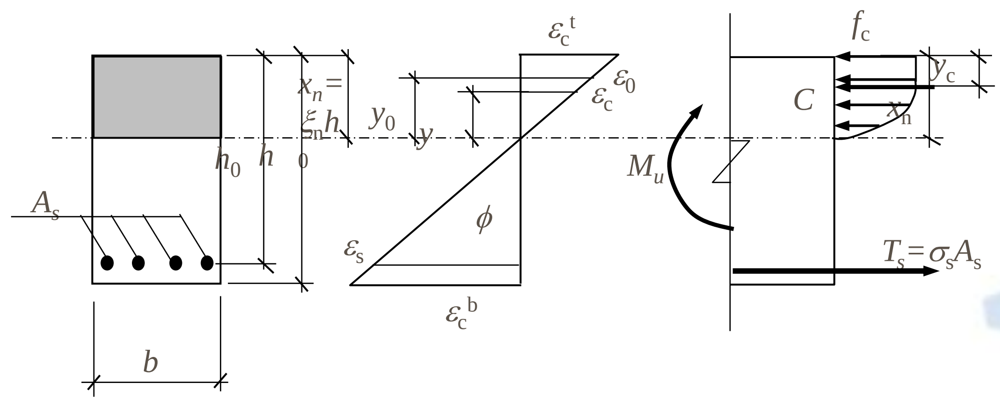
$$M_u = 0.798 f_c \xi_n b h_0^2 (1-0.412\xi_n) = \sigma_s A_s h_0 (1-0.412\xi_n) \qquad (\sigma_s \le f_y)$$

➢受弯构件受力分析

- 破坏阶段

对适筋梁，达极限状态时，

$$\varepsilon_c^t = \varepsilon_{cu} = 0.0033, \sigma_s = f_y$$

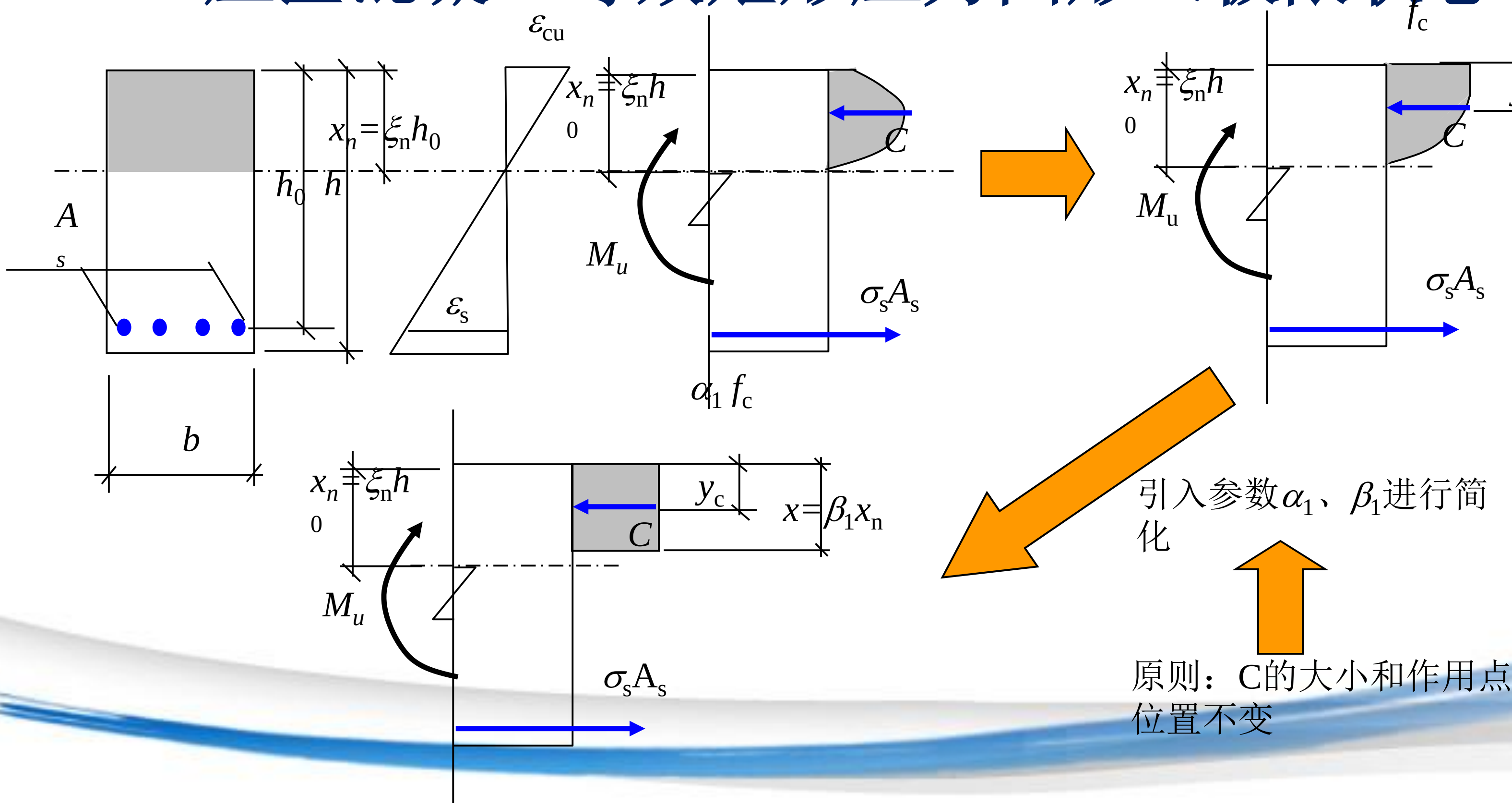


$$\sum X = 0 \Rightarrow \xi_n = 1.253\rho_s \frac{f_y}{f_c}$$

$$\sum M = 0 \Rightarrow M_u = f_y A_s h_0 (1 - 0.412\xi_n) = f_c b h_0^2 \xi_n (0.798 - 0.329\xi_n)$$

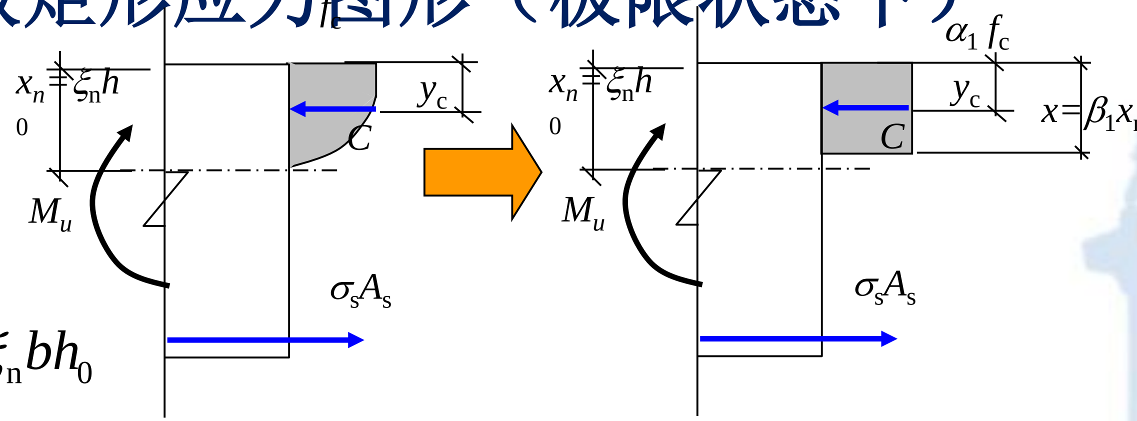
➢受弯构件正截面简化分析

- 压区混凝土等效矩形应力图形（极限状态下）



➢受弯构件正截面简化分析

- 压区混凝土等效矩形应力图形（极限状态下）



由C的大小不变

$$C=\alpha_1 f_c b\xi_n h_0\left(1-\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{cu}}\right)=\alpha_1 f_c\beta_1\xi_n bh_0$$

$$\alpha_1=\frac{1}{\beta_1}\left(1-\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{cu}}\right)$$

由C的位置不变

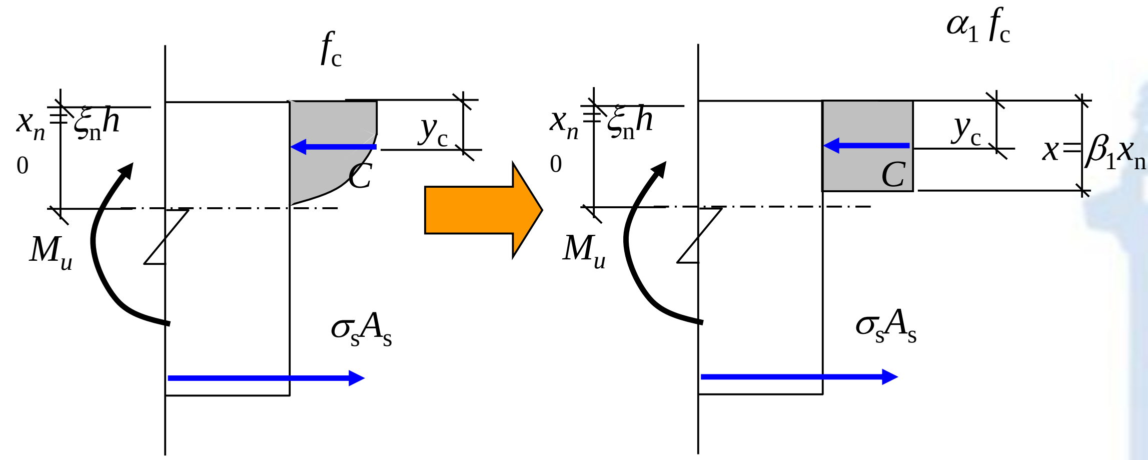
$$y_c=\xi_n h_0\left(1-\frac{\frac{1}{2}-\frac{1}{12}\left(\frac{\varepsilon_0}{\varepsilon_{cu}}\right)^2}{1-\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{cu}}}\right)=0.5\beta_1\xi_n h_0,\ \beta_1=\frac{1-\frac{2}{3}\frac{\varepsilon_0}{\varepsilon_{cu}}+\frac{1}{6}\left(\frac{\varepsilon_0}{\varepsilon_{cu}}\right)^2}{1-\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{cu}}}$$

➢受弯构件正截面简化分析

- 压区混凝土等效矩形应力图形（极限状态下）

$$\alpha_1=\frac{1}{\beta_1}\left(1-\frac{1}{3}\frac{\varepsilon_{c0}}{\varepsilon_{cu}}\right)$$

$$\beta_1=\frac{1-\frac{2}{3}\frac{\varepsilon_0}{\varepsilon_{cu}}+\frac{1}{6}\left(\frac{\varepsilon_0}{\varepsilon_{cu}}\right)^2}{1-\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{cu}}}$$



当$f_{cu}\le 50$Mpa时，

$\varepsilon_0=0.002,\varepsilon_{cu}=0.0033$

$$\begin{cases}\alpha_1=0.969\\ \beta_1=0.824\end{cases}$$

$$\begin{cases}\alpha_1=1.0,\beta_1=0.8 & f_{cu}\le 50\text{Mpa}\\ \alpha_1=0.94,\beta_1=0.74, & f_{cu}=80\text{Mpa}\end{cases}$$

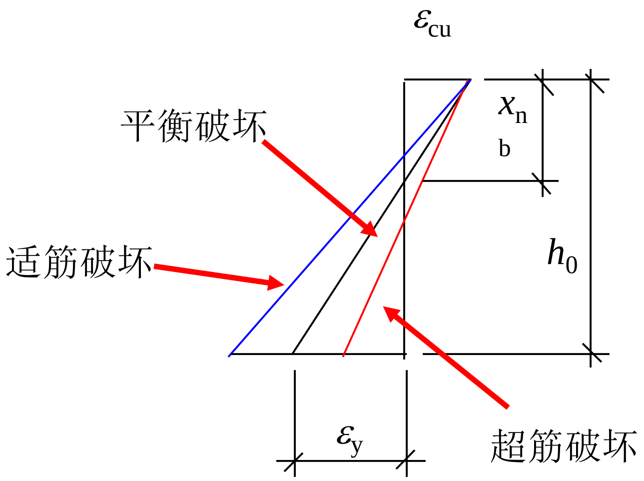
线性插值（《混凝土结构设计规范》GB50010 ）

➢受弯构件正截面简化分析

- 界限受压区高度

x_{nb} = 界限受压区高度

ξ_{nb} = 界限受压区相对高度



$$\xi_{nb} = \frac{x_{nb}}{h_0} = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_y}$$

x_b = 矩形应力图形的界限受压区高度

ξ_b = 矩形应力图形的界限受压区相对高度

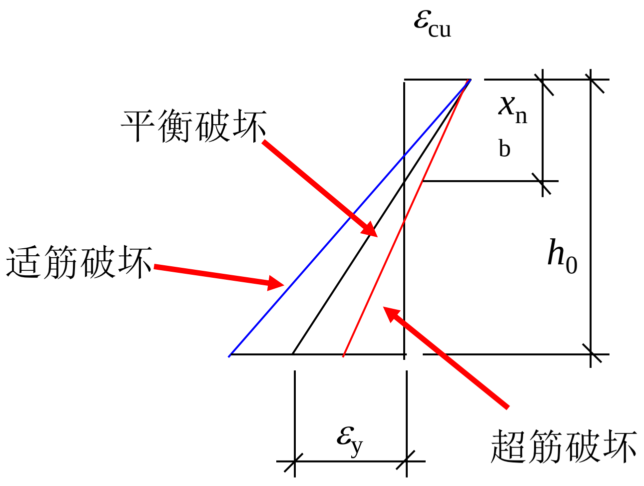
$$\xi_b = \frac{x_{nb}}{h_0} = \frac{\beta_1 x_b}{h_0} = \frac{\beta_1 \varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_y} = \frac{\beta_1}{1 + \dfrac{\varepsilon_y}{\varepsilon_{cu}}} = \frac{\beta_1}{1 + \dfrac{f_y}{E_s \varepsilon_{cu}}}$$

➢受弯构件正截面简化分析

- 界限受压区高度

$f_{cu} \leq 50\text{Mpa}$时：

$$\xi_b = \frac{0.8}{1 + \dfrac{f_y}{0.0033E_s}}$$



$\xi < \xi_b$即$\xi_n < \xi_{nb}$ ➡ 适筋梁

$\xi = \xi_b$即$\xi_n = \xi_{nb}$ ➡ 平衡配筋梁

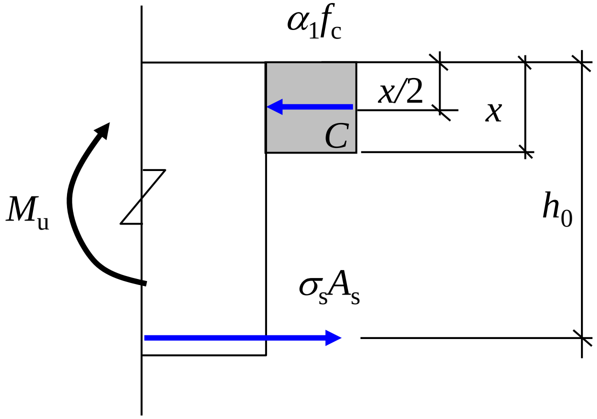
$\xi > \xi_b$即$\xi_n > \xi_{nb}$ ➡ 超筋梁

➢受弯构件正截面简化分析

• 极限受弯承载力

基本公式

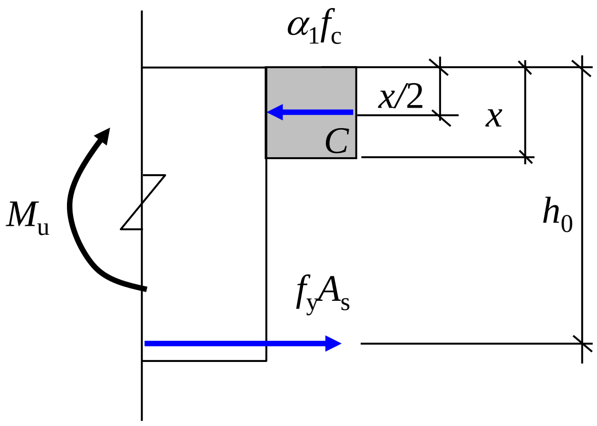
$$
\begin{cases}
\alpha_1 f_c bx = \sigma_s A_s \\
M_u = \alpha_1 f_c bx(h_0 - \dfrac{x}{2}) = \sigma_s A_s (h_0 - \dfrac{x}{2})
\end{cases}
$$



受弯构件正截面简化分析

• 极限受弯承载力

适筋梁



$$\begin{cases} \alpha_1 f_c b x = f_y A_s \\ M_u = \alpha_1 f_c b x (h_0 - \dfrac{x}{2}) = f_y A_s (h_0 - \dfrac{x}{2}) \end{cases}$$

$$\xi = \frac{x}{h_0} = \frac{f_y A_s}{\alpha_1 f_c b h_0} = \rho \frac{f_y}{\alpha_1 f_c}$$

截面抵抗矩系数

$$M_u = \alpha_1 f_c b h_0^2 \xi (1 - 0.5\xi) = \alpha_s \alpha_1 f_c b h_0^2$$

$$= A_s f_y h_0 (1 - 0.5\xi) = A_s f_y \gamma_s h_0$$

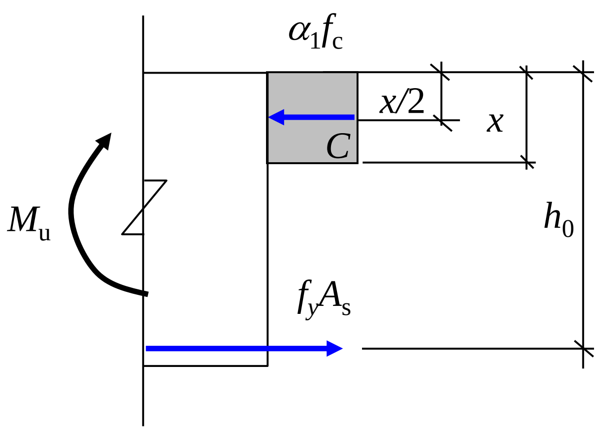
截面内力臂系数

将ξ、α_s、γ_s制成表格，知道其中一个可查得另外两个

➢受弯构件正截面简化分析

• 极限受弯承载力

适筋梁的最大配筋率（平衡配筋梁的配筋率）



$$\rho_{max} = \rho_b = \xi_b \frac{\alpha_1 f_c}{f_y}$$

$$\alpha_{max} = \xi_b (1 - 0.5\xi_b)$$

$$M_{u,max} = \alpha_1 f_c b h_0^2 \xi_b (1 - 0.5\xi_b) = \alpha_{s,max} \alpha_1 f_c b h_0^2$$

《混凝土结构设计规范》GB50010中各种钢筋所对应的ξ_b、$\alpha_{s,max}$、列于教材表5-1中

保证不发生超筋破坏

$\xi \le \xi_b$或

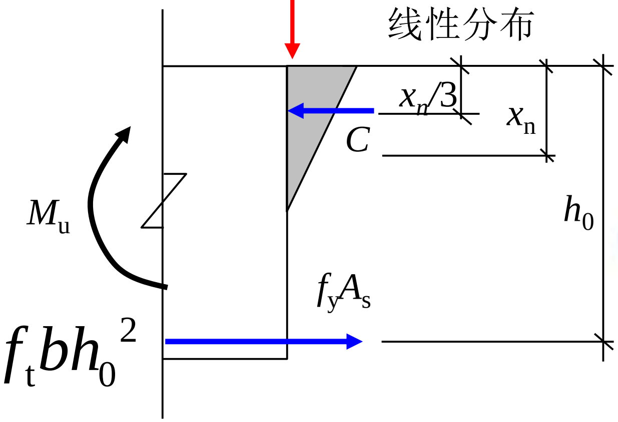
$\alpha_s \le \alpha_{s,max}$或

$M_u \le M_{u,max}$

➢受弯构件正截面简化分析

• 极限受弯承载力

适筋梁的最小配筋率



钢筋混凝土梁的M_y=素混凝土梁的受弯承载力M_{cr}

$$M_{cr} = 0.292 f_t bh^2 = 0.292 f_t b\left(1.05h_0\right)^2 = 0.322 f_t bh_0^{\ 2}$$

$$M_y \approx f_y A_s (h_0 - \frac{x_n}{3}) \approx f_y A_s \times 0.9h_0$$

$$\rho_{min} = \frac{A_s}{bh_0} = 0.36\frac{f_t}{f_y}$$

偏于安全地

$$\rho_{min} = 0.45\frac{f_t}{f_y}$$

具体应用时，应根据不同情况，进行调整

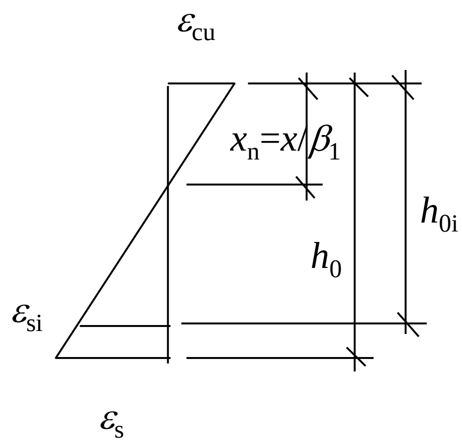
《混凝土结构设计规范》GB50010中取：A_{smin}=$\rho_{smin}bh$

➢受弯构件正截面简化分析

• 极限受弯承载力

超筋梁的极限承载力

关键在于求出钢筋的应力



任意位置处钢筋的应变和应力

$$\varepsilon_{si}=\frac{h_{0i}-x_n}{x_n}\varepsilon_{cu}=\varepsilon_{cu}(\frac{h_{0i}\beta_1}{x}-1)=\varepsilon_{cu}(\frac{h_{0i}\beta_1}{\xi h_0}-1)$$

$$\sigma_{si}=E_s\varepsilon_{cu}(\frac{h_{0i}\beta_1}{\xi h_0}-1)$$

只有一排钢筋

$$\sigma_s=E_s\varepsilon_{cu}(\frac{\beta_1}{\xi}-1)$$

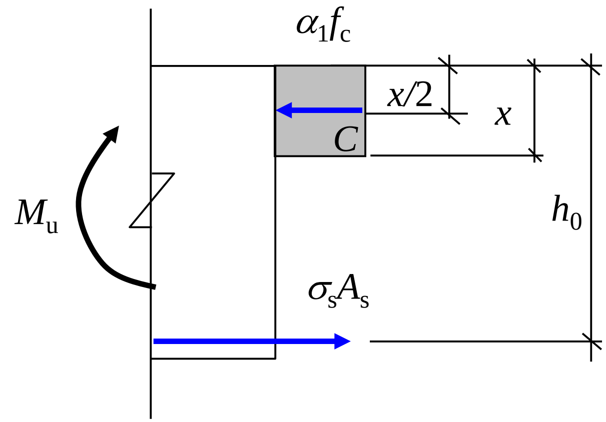
$f_{cu}\le 50$Mpa

$$\sigma_s=0.0033E_s(\frac{0.8}{\xi}-1)$$

➢受弯构件正截面简化分析

• 极限受弯承载力

超筋梁的极限承载力



$$
\begin{cases}
\alpha_1 f_c bx = \sigma_s A_s \\
M_u = \alpha_1 f_c bx(h_0 - \dfrac{x}{2}) = \sigma_s A_s (h_0 - \dfrac{x}{2}) \\
\sigma_s = 0.0033 E_s \left(\dfrac{0.8}{\xi} - 1 \right)
\end{cases}
$$

避免求解高次方程

作简化

$$\sigma_s = f_y \frac{\xi - 0.8}{\xi_b - 0.8}$$

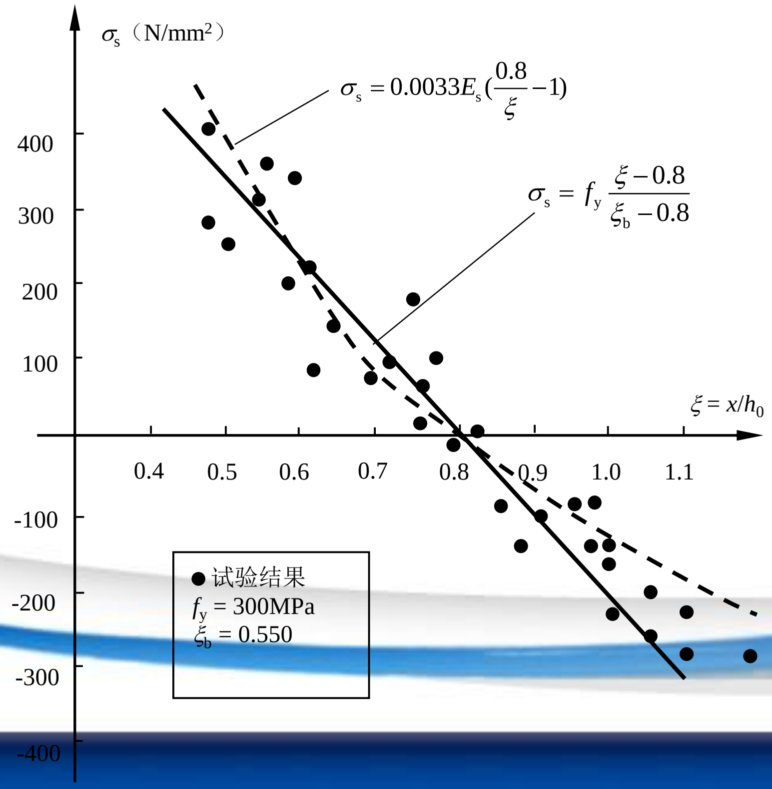
解方程可求出M_u

➢受弯构件正截面简化分析

• 极限受弯承载力

超筋梁的极限承载力

$$\sigma_s = 0.0033E_s\left(\frac{0.8}{\xi} - 1\right)$$

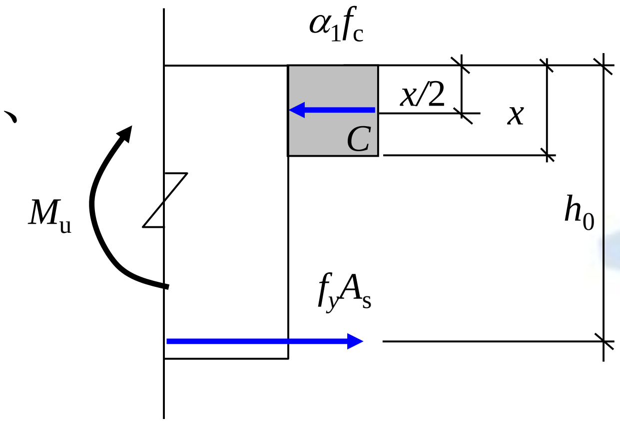


$$\sigma_s = f_y \frac{\xi - 0.8}{\xi_b - 0.8}$$

➤受弯构件正截面简化分析

• 承载力公式的应用

既有构件正截面抗弯承载力（已知b、h_0、f_y、A_s，求M_u）



$$\rho = \frac{A_s}{bh_0}, \left(\frac{A_s}{bh}\right)$$

$< \rho_{min}$

素混凝土梁的受弯承载力M_{cr}

$\rho_{min} \le \rho \le \rho_b$

适筋梁的受弯承载力M_u

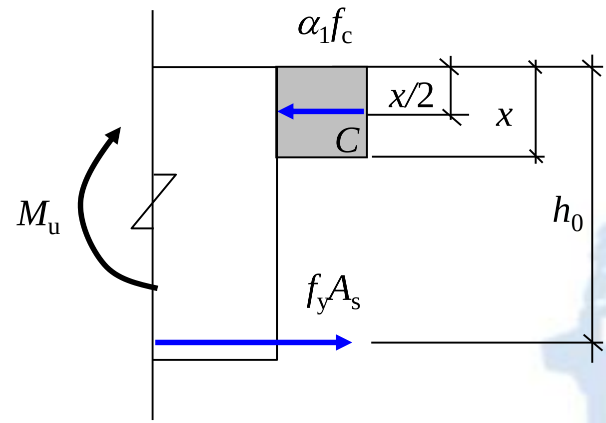
$> \rho_b$

超筋梁的受弯承载力M_u

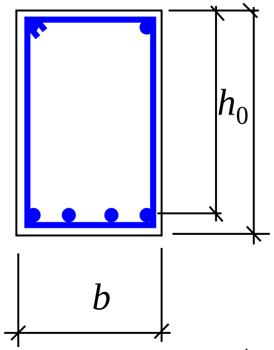
➢受弯构件正截面简化分析

• 承载力公式的应用

既有构件正截面抗弯承载力（已知b,h_0, f_y ,A_s，求M_u）

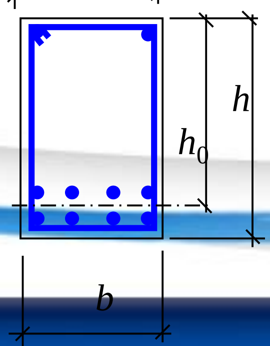


当采用单排钢筋时



$$h_0 = h - c - d/2$$

当采用双排钢筋时



$$h_0 = h - [c + d + \max(25/2, d/2)]$$

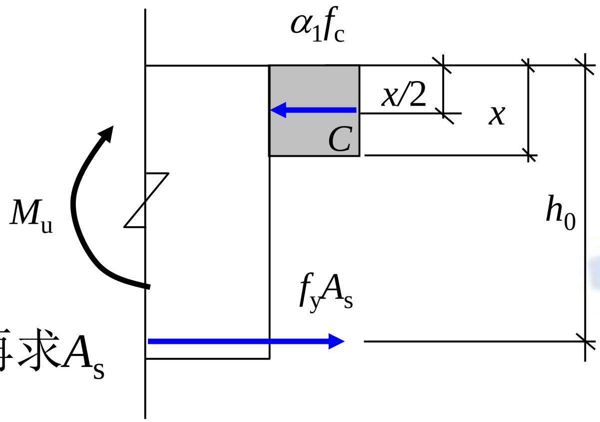
➢受弯构件正截面简化分析

- 承载力公式的应用

基于承载力的截面设计（已知b、h_0、f_y、 M，求A_s ）

$$\begin{cases} \alpha_1 f_c bx = f_y A_s \\ M = M_u = \alpha_1 f_c bx(h_0 - \dfrac{x}{2}) = f_y A_s (h_0 - \dfrac{x}{2}) \end{cases}$$

先求x再求A_s



$$\rho = \frac{A_s}{bh_0}, \left(\frac{A_s}{bh}\right)$$

$\xi > \xi_b$

加大截面尺寸重新进行设计(**或先求出$M_{u,\ max}$，若$M > M_{u,\ max}$，加大截面尺寸重新进行设计**）

$\rho_{min} \le \rho$

$\xi \le \xi_b$

OK！

$< \rho_{min}$

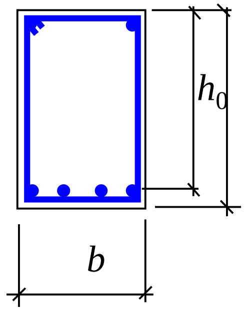
$$A_s = \rho_{min} bh$$

➤受弯构件正截面简化分析

• 承载力公式的应用

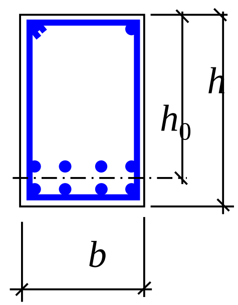
基于承载力的截面设计（已知b、h_0、f_y、 M， 求A_s ）

当采用单排钢筋时



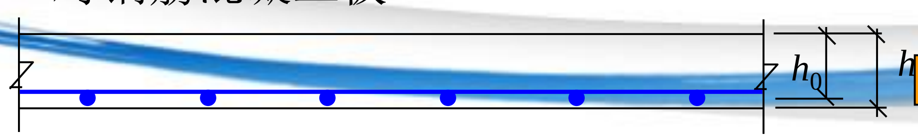
$$h_0 = h - 35(\mathrm{mm})$$

当采用双排钢筋时



$$h_0 = h - 60(\mathrm{mm})$$

对钢筋混凝土板

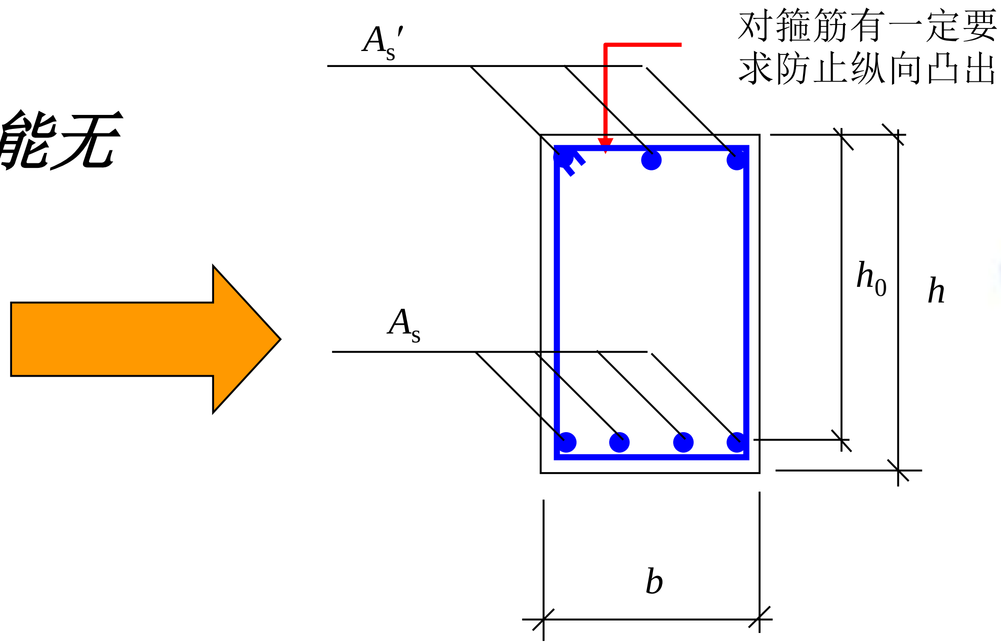


$$h_0 = h - 20(\mathrm{mm})$$

➢双筋矩形截面受弯构件

截面的弯矩较大，高度不能无限制地增加

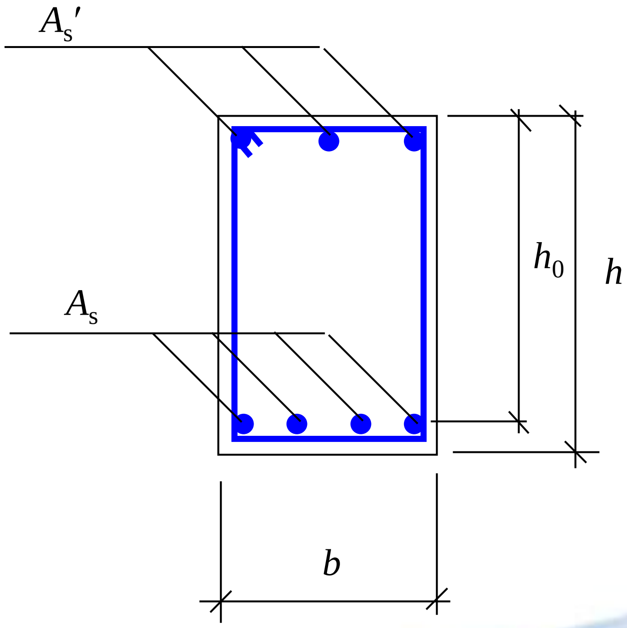
截面承受正、负变化的弯矩



➢双筋矩形截面受弯构件

- 试验研究

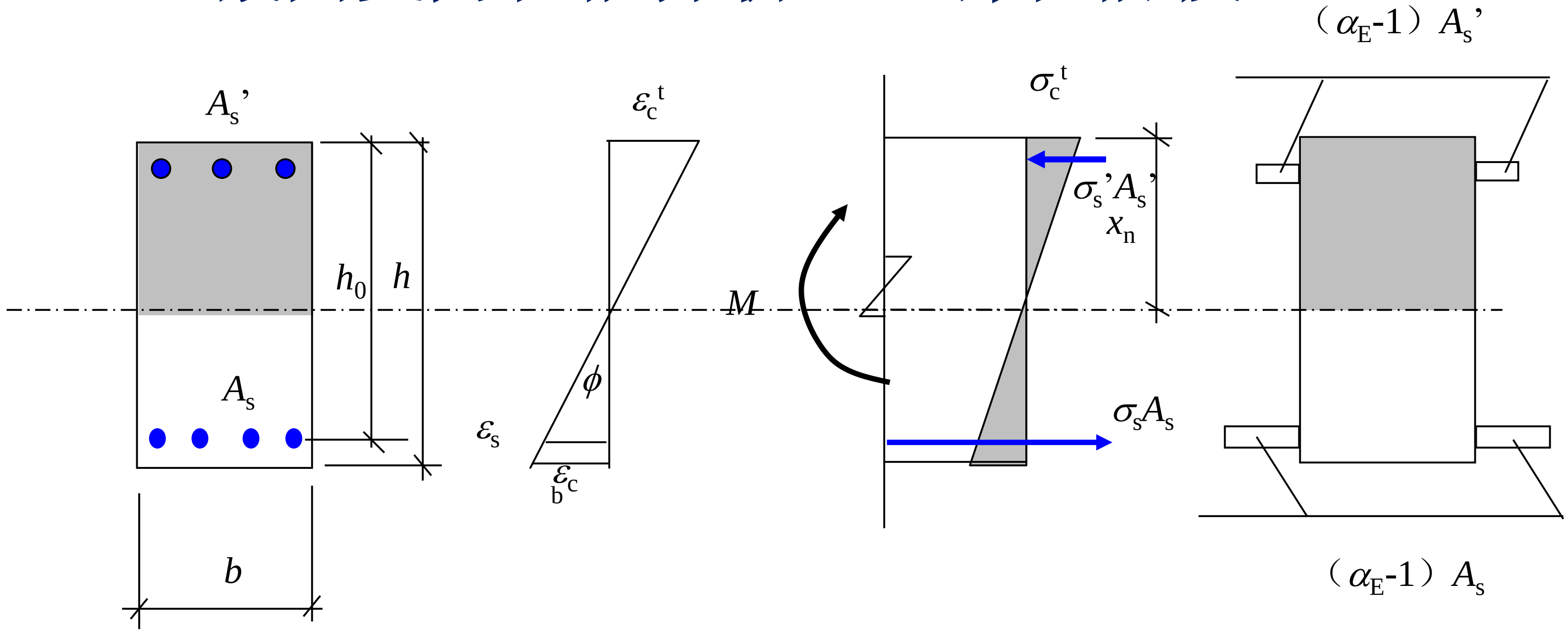
由于钢筋配置较多，一般不会发生少筋破坏



和单筋矩形截面受弯构件类似分三个工作阶段

➢双筋矩形截面受弯构件

- 正截面受力性能分析——弹性阶段



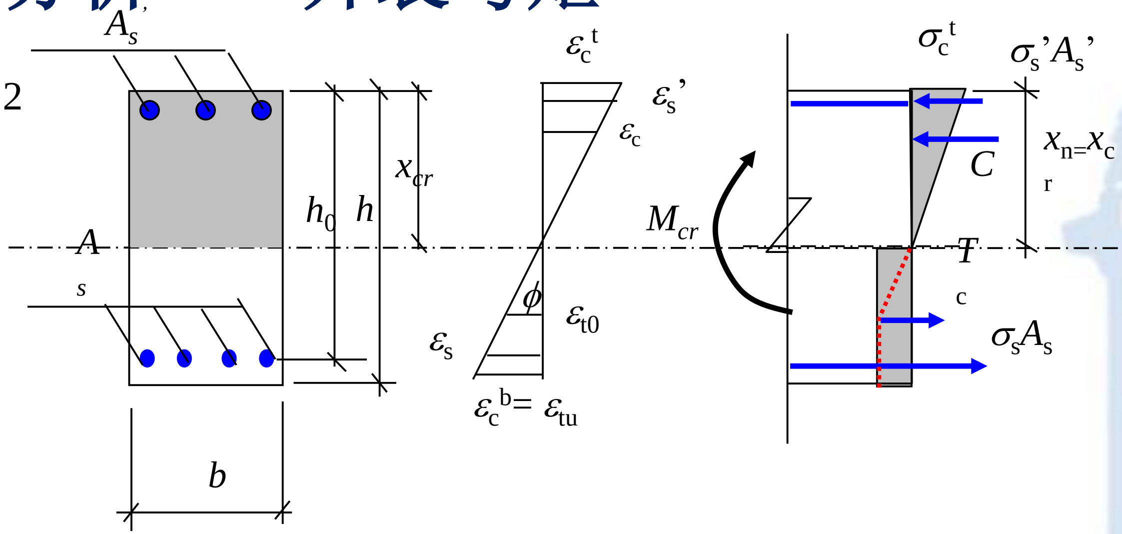
用材料力学的方法按换算截面进行求解

➢双筋矩形截面受弯构件

- 正截面受力性能分析——开裂弯矩

$$M_{\mathrm{cr}}=0.292(1+2.5\alpha_A)f_{\mathrm{t}}bh^2$$

$$+\sigma_{\mathrm{s}}'A_{\mathrm{s}}'(\frac{1}{3}x_{\mathrm{cr}}-a_{\mathrm{s}}')$$



$$\varepsilon_{\mathrm{s}}'=\frac{x_{\mathrm{cr}}-a_{\mathrm{s}}'}{h-x_{\mathrm{cr}}}\varepsilon_{\mathrm{tu}}$$

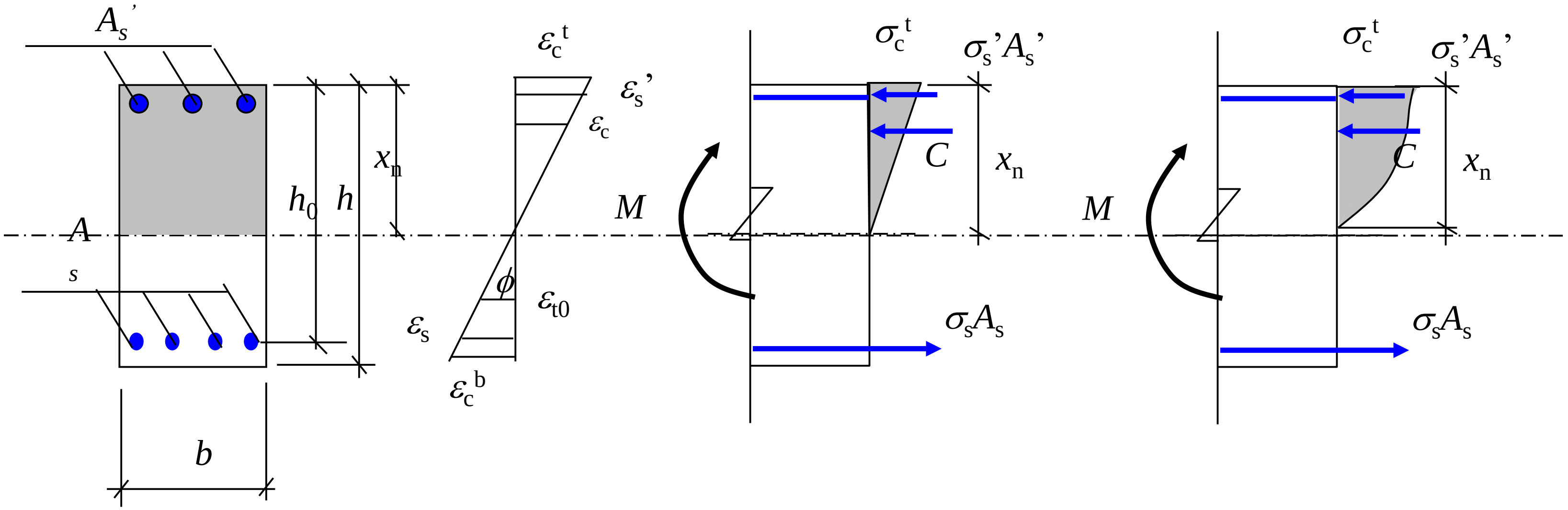
$$=2\frac{x_{\mathrm{cr}}-a_{\mathrm{s}}'}{h-x_{\mathrm{cr}}}\frac{f_{\mathrm{t}}}{E_{\mathrm{c}}}$$

$$M_{\mathrm{cr}}=0.292(1+2.5\alpha_A+0.25\alpha_A')f_{\mathrm{t}}bh^2$$

$$\alpha_A'=2\alpha_{\mathrm{E}}(A_{\mathrm{s}}'/bh)$$

➢双筋矩形截面受弯构件

• 正截面受力性能分析——带裂缝工作



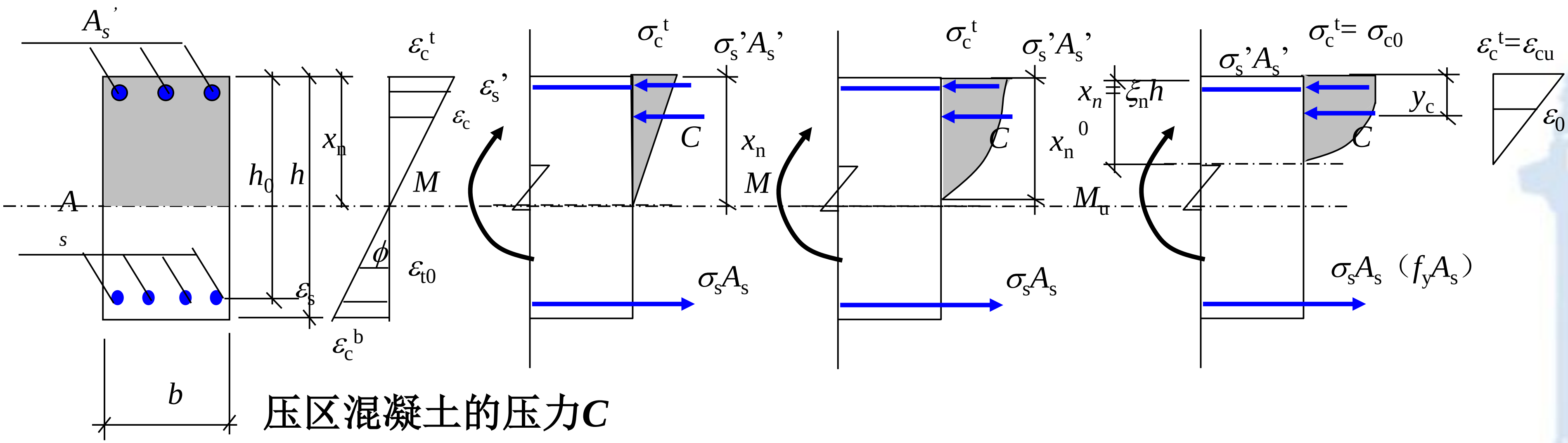
荷载较小时，混凝土的应力可简化为直线型分布

荷载增大时，混凝土的应力由为直线型分布转化为曲线型分布

和单筋矩形截面梁类似

➢双筋矩形截面受弯构件

• 正截面受力性能分析——破坏阶段



压区混凝土的压力*C*

和单筋矩形截面梁的受压区相同

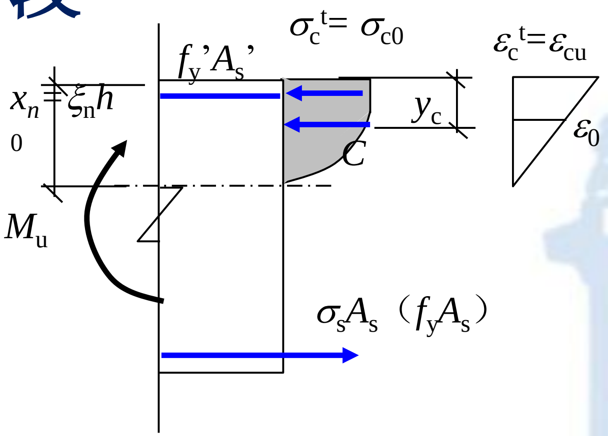
***C*的作用位置y_c**

➢双筋矩形截面受弯构件

• 正截面受力性能分析——破坏阶段

当f_{cu}≤50Mpa时，根据平截面假定有：

$$\sigma_s^{'} = 0.0033E_s(\frac{a_s^{'}}{x_n} - 1)$$



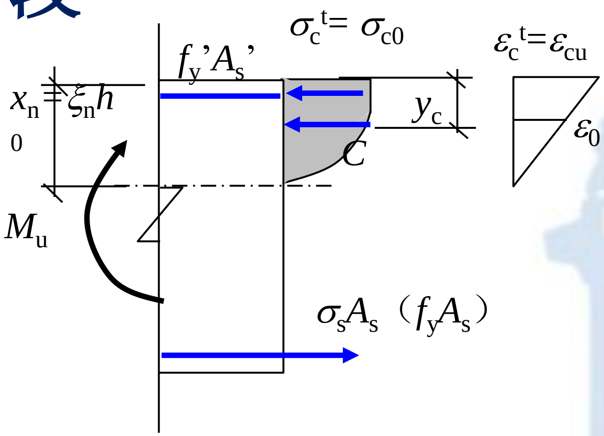
以E_s=2×10^{5}Mpa，a_s’=0.5×0.8x_n代入上式，则有：σ_s’=-396Mpa

结论：

当x_n≥2 a_s’/0.8 时，HPB300、HRB335、HRB400及RRB400钢均能受压屈服

➢双筋矩形截面受弯构件

• 正截面受力性能分析——破坏阶段



当f_{cu}≤50Mpa时，根据平衡条件则有：

$$\xi_n = 1.253\left(\rho_s \frac{f_y}{\sigma_{c0}} - \rho_s' \frac{f_y}{\sigma_{c0}}\right)$$

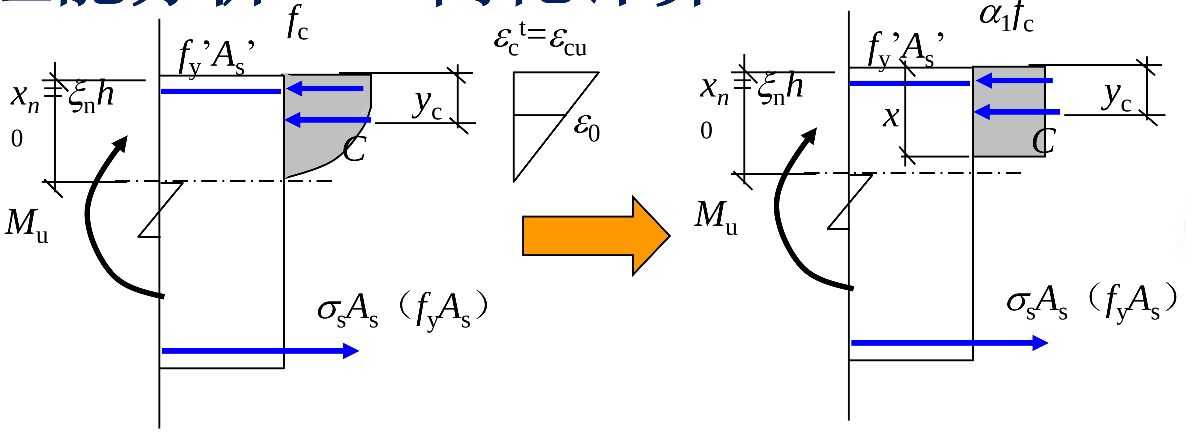
$$M_u = f_y A_s h_0 (1 - 0.412\xi_n) + f_y' A_s' h_0 \left(0.412\xi_n - \frac{a_s'}{h_0}\right)$$

$$= \sigma_{c0} b h_0^2 \xi_n (0.798 - 0.329\xi_n) + f_y' A_s' h_0 \left(1 - \frac{a_s'}{h_0}\right)$$

➤双筋矩形截面受弯构件

- 正截面受力性能分析——简化计算

β_1、α_1的计算方法和单筋矩形截面梁相同

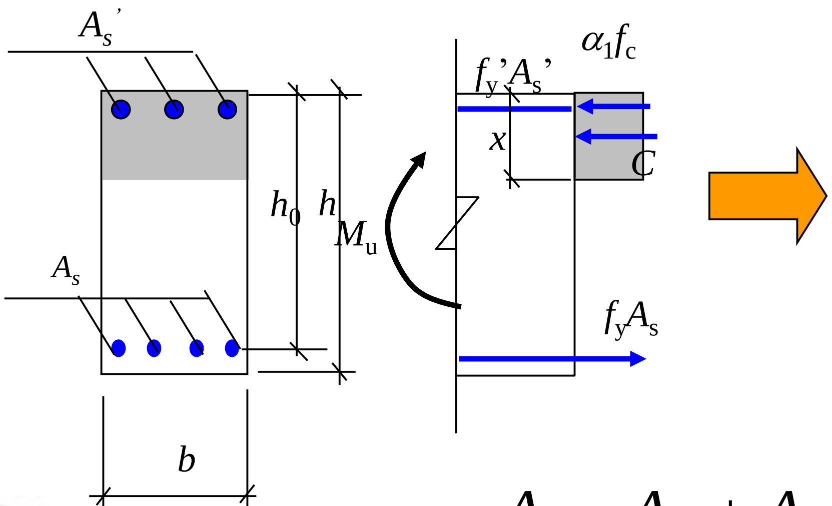


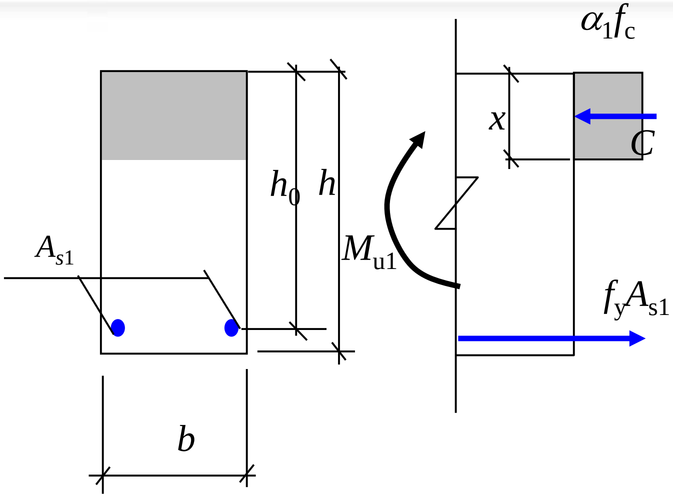
$$
\begin{cases}
\alpha_1 f_c bx + f_y' A_s' = f_y A_s \\
M_u = \alpha_1 f_c bx\left(h_0 - \dfrac{x}{2}\right) + f_y' A_s' (h_0 - a_s')
\end{cases}
$$

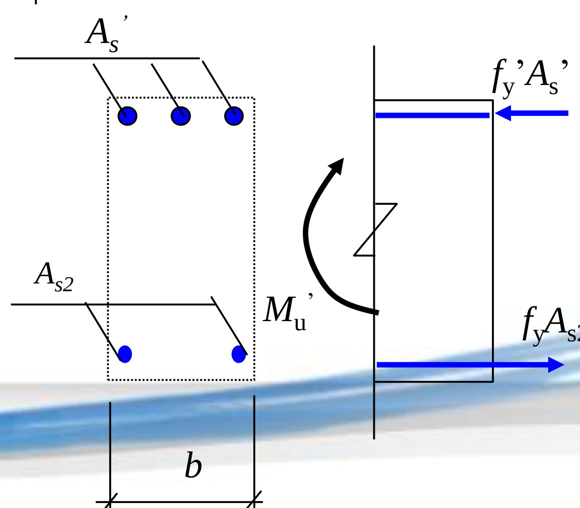
➢双筋矩形截面受弯构件

- 正截面受力性能分析

简化计算







$$A_s = A_{s1} + A_{s2}$$

➢双筋矩形截面受弯构件

• 正截面受力性能分析——简化计算

承载力公式的适用条件

1. 保证不发生少筋破坏: $\rho > \rho_{\min}$ (可自动满足)

2. 保证不发生超筋破坏:

$$\xi = \frac{x}{h_0} \le \xi_b \text{,或}$$

$$\rho_1 = \frac{A_{s1}}{bh_0} \le \rho_{\max} = \xi_b \frac{\alpha_1 f_c}{f_y} \text{,或}$$

$$M_1 \le \alpha_{s,\max} \alpha_1 f_c b h_0^2$$

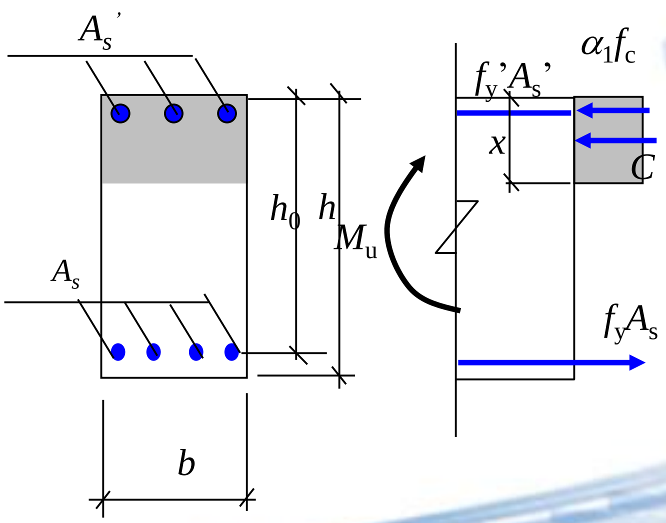
➢双筋矩形截面受弯构件

• 正截面受力性能分析——简化计算

承载力公式的适用条件

3. 保证受压钢筋屈服: $x>2a_s'$，当该条件不满足时，应按下式求承载力

$$
\begin{cases}
\alpha_1 f_c bx + \sigma_s' A_s' = f_y A_s \\
M_u = \alpha_1 f_c bx(h_0 - \dfrac{x}{2}) + |\sigma_s'| A_s'(h_0 - a_s') \\
\sigma_s' = E_s \varepsilon_{cu} (\dfrac{\beta_1 a_s'}{\xi h_0} - 1)
\end{cases}
$$



或近似取 $x=2a_s'$ 则， $M_u = f_y A_s h_0 (1 - \dfrac{a_s'}{h_0})$

➢双筋矩形截面受弯构件

- **承载力公式的应用**

既有构件正截面抗弯承载力

$$A_{s2}=A_s'f_y'/f_y,\ A_{s1}=A_s-A_{s2}$$

$$M_u'=f_y'A_s'(h_0-a_s')$$

求*x*

$<2a_s'$

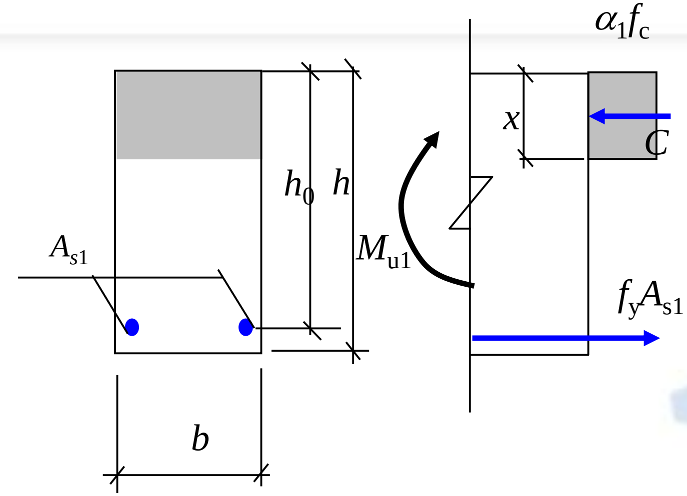
$$M_u=f_yA_sh_0\left(1-\frac{a_s'}{h_0}\right)$$

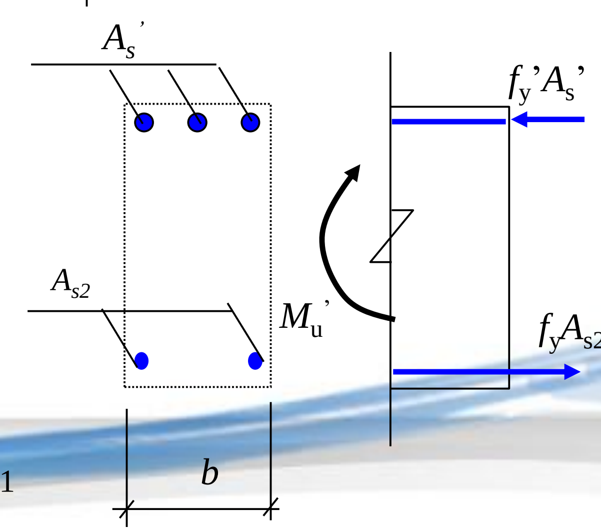
$2a_s'\le x\le \xi_b h_0$

适筋梁的受弯承载力M_{u1}

$>\xi_b h_0$

超筋梁的受弯承载力M_{u1}

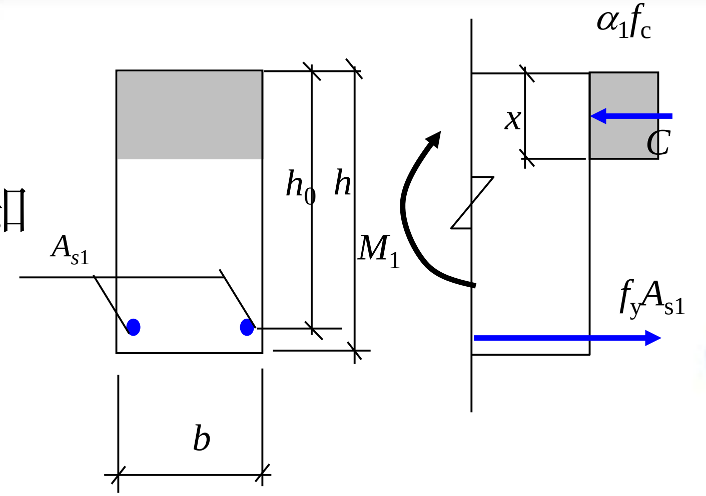




➢双筋矩形截面受弯构件

• 承载力公式的应用

基于承载力的构件截面设计I——A_s’未知



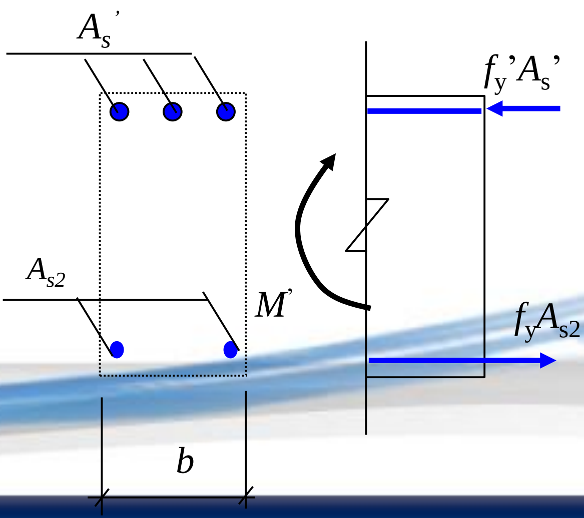
$$x = \xi_b h_0$$

⇩

$$A_{s1} = \alpha_1 f_c b x / f_y, M_1 = A_{s1} f_y (h_0 - 0.5x)$$

⇩

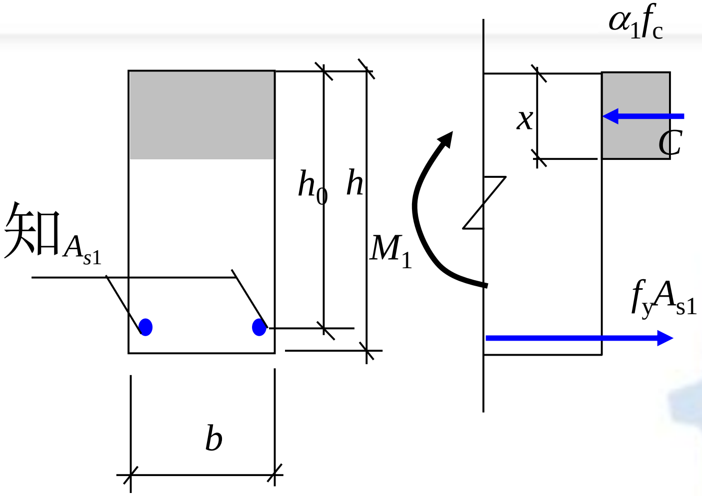
$$M' = M - M_1, A_{s2} = M' / (h_0 - a_s') f_y, A_s' = A_{s2} f_y / f_y'$$



➢双筋矩形截面受弯构件

• 承载力公式的应用

基于承载力的构件截面设计II——A_s’已知



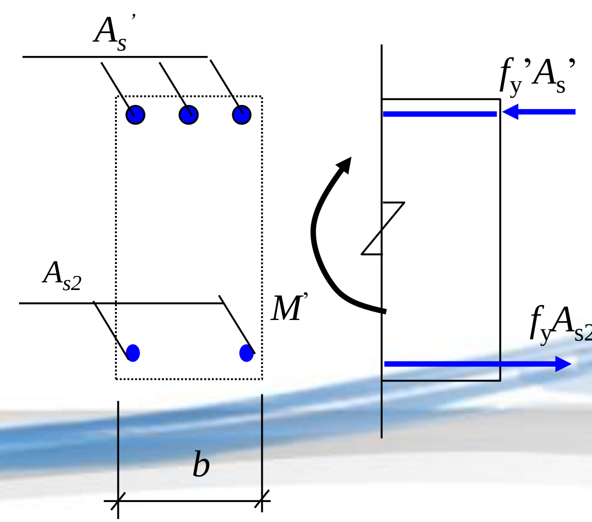
$$A_{s2}=A_s' f_y' / f_y, M'=A_{s2} f_y (h_0-a_s')$$

$$M_1 = M - M', \text{求} x$$

$< 2a_s'$：按单筋截面适筋梁求A_s,但应进行最小配筋率验算

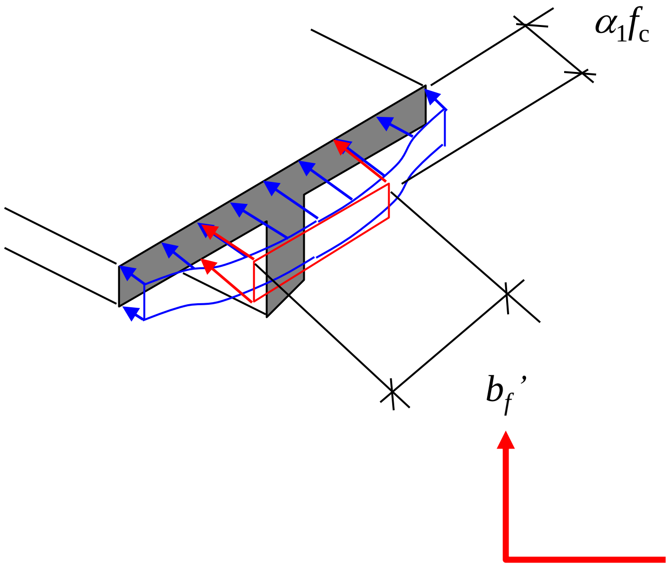
$2a_s' \le x \le \xi_b h_0$：按适筋梁求$A_{s1}$

$> \xi_b h_0$：按A_s’未知重新求A_s’和A_s



➢T形截面受弯构件

- 翼缘的计算宽度

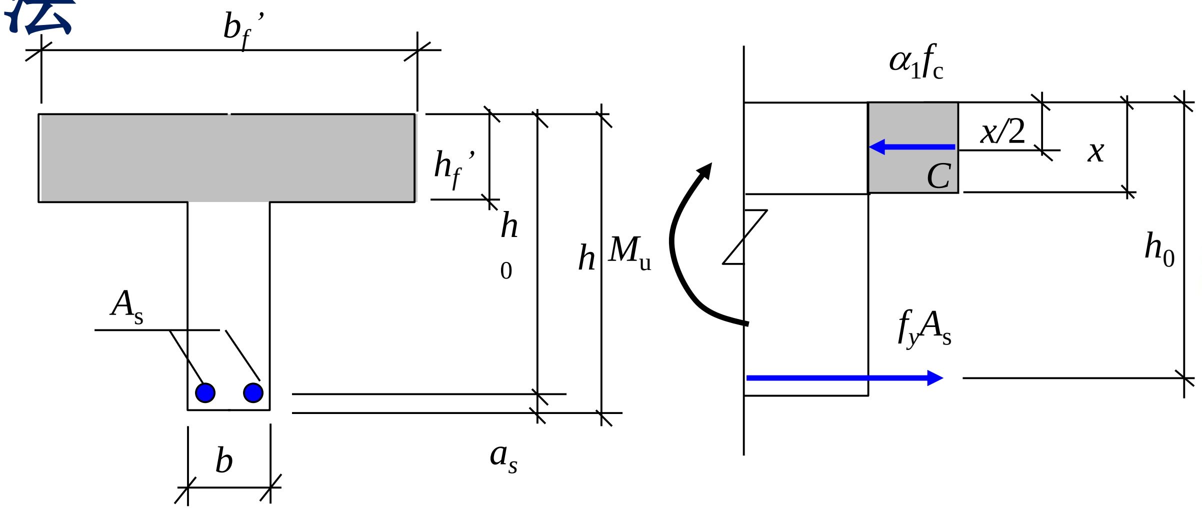


见教材表5-2

➢T形截面受弯构件

• 简化计算方法

两类T形截面判别



$$f_y A_s \le \alpha_1 f_c b_f' h_f'$$,或

$$M \le \alpha_1 f_c b_f' h_f' (h_0 - \frac{h_f'}{2})$$

I类 否则 II类

中和轴位于翼缘

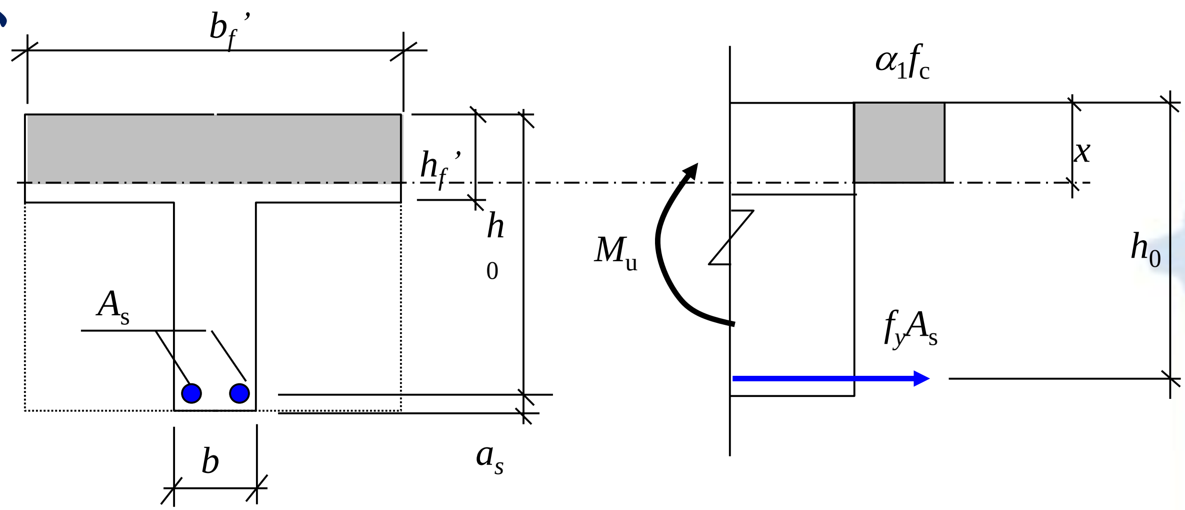
中和轴位于腹板

➢T形截面受弯构件

- 简化计算方法

I类T形截面

按$b_f{}'\times h$的矩形截面计算



$$\begin{cases} \alpha_1 f_c b x = f_y A_s \\ M_u = \alpha_1 f_c b_f' x\left(h_0 - \dfrac{x}{2}\right) = f_y A_s\left(h_0 - \dfrac{x}{2}\right) \end{cases}$$

$$\xi \le \xi_b$$

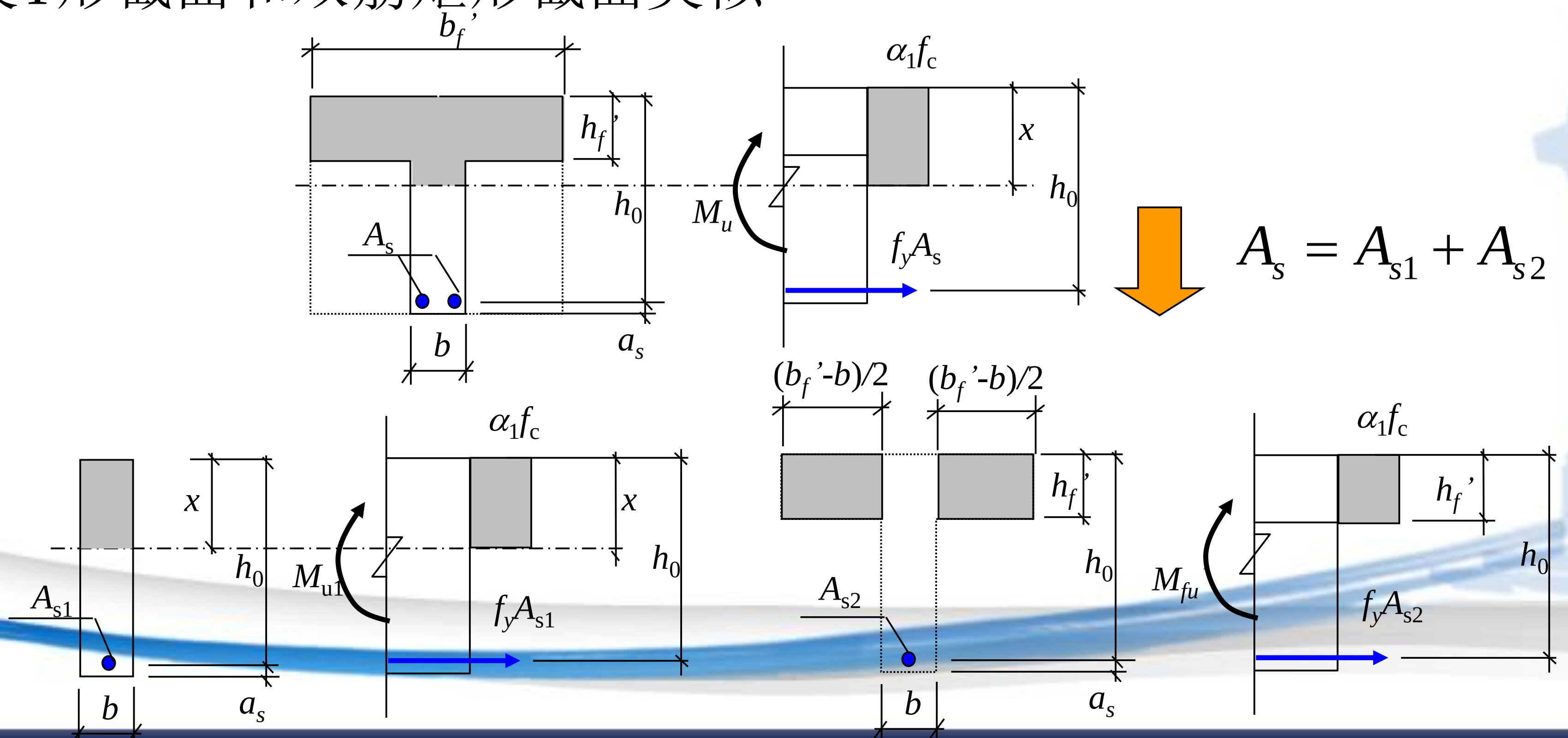
$$\rho = \frac{A_s}{bh} \ge \rho_{min}$$

T形截面开裂弯矩同截面为腹板的矩形截面的开裂弯矩几乎相同

➢T形截面受弯构件

• 简化计算方法

II类T形截面和双筋矩形截面类似

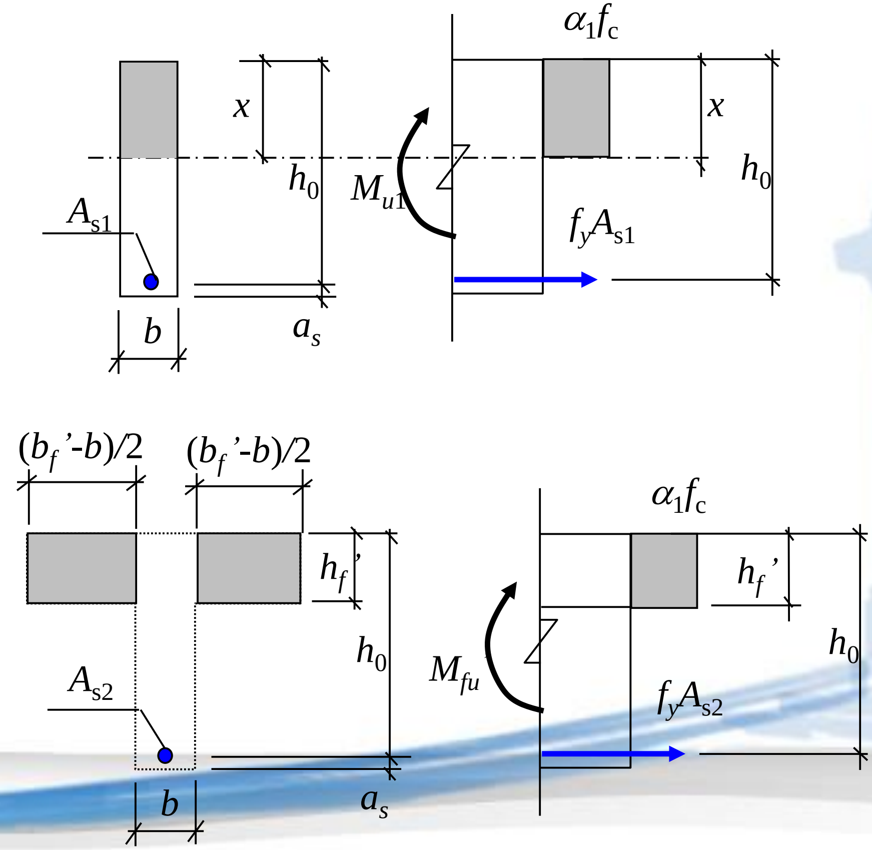


➢T形截面受弯构件

- 简化计算方法

II类T形截面和双筋矩形截面类似

$$
\begin{cases}
\alpha_1 f_c bx + \alpha_1 f_c (b_f' - b)h_f' = f_y A_s \\
M_u = M_{u1} + M_{fu}' = \alpha_1 f_c bx(h_0 - \dfrac{x}{2}) \\
\qquad + \alpha_1 f_c (b_f' - b)h_f' (h_0 - \dfrac{h_f'}{2})
\end{cases}
$$



➢T形截面受弯构件

• 简化计算方法

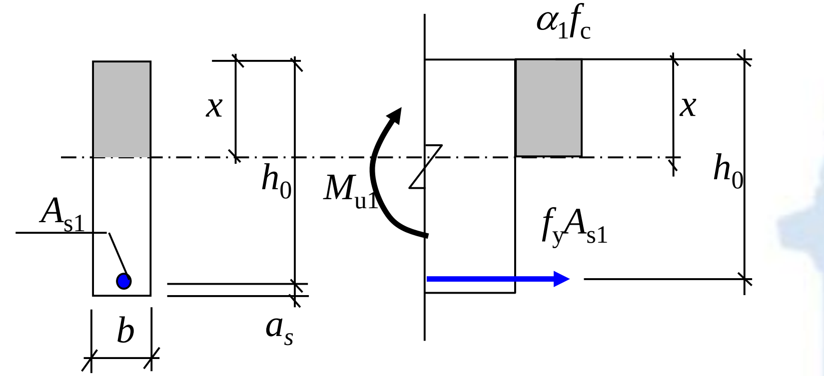
II类T形截面和双筋矩形截面类似

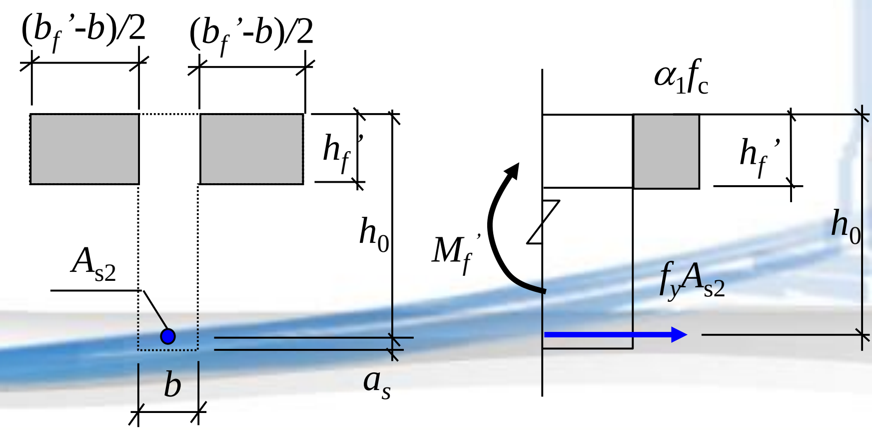
$\rho \geq \rho_{\min}$，一般可自动满足，但需要验算

$$\xi = \frac{x}{h_0} \leq \xi_b, \text{或}$$

$$\rho_{s1} = \frac{A_{s1}}{bh_0} \leq \rho_{s,\max} = \xi_b \frac{\alpha_1 f_c}{f_y}, \text{或}$$

$$M_1 \leq \alpha_{s,\max} \alpha_1 f_c b h_0^2$$



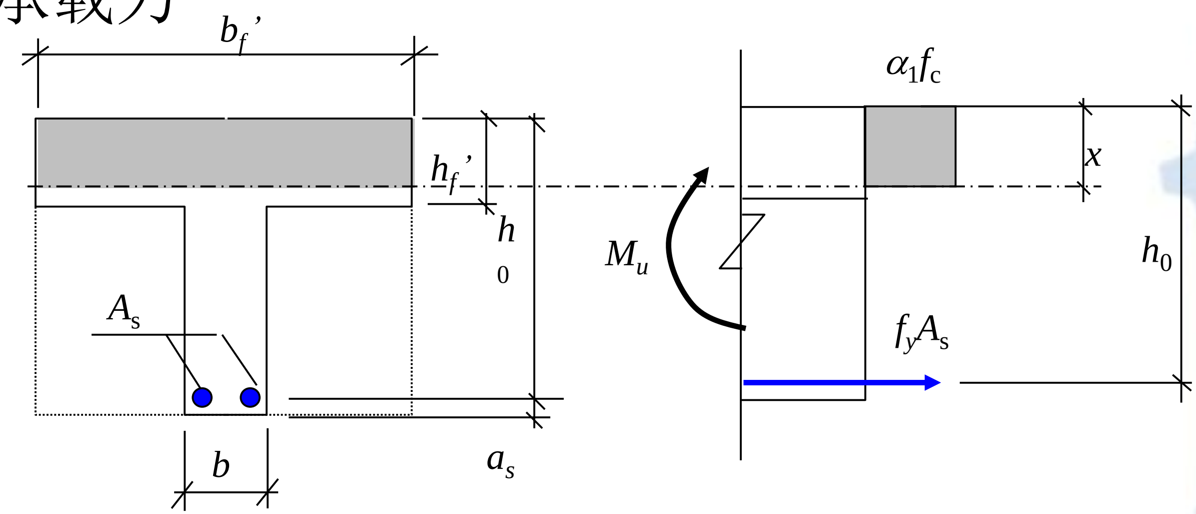


➢T形截面受弯构件

- 简化计算方法

既有构件正截面抗弯承载力

$$f_y A_s \le \alpha_1 f_c b_f' h_f'$$



I类T形截面

按$b_f'\times h$的矩形截面计算构件的承载力

若$A_s < \rho_{\min} bh$

按$b\times h$的矩形截面的开裂弯矩计算构件的承载力

➢T形截面受弯构件

- **简化计算方法**

既有构件正截面抗弯承载力

$$f_y A_s > \alpha_1 f_c b_f' h_f'$$

⇩

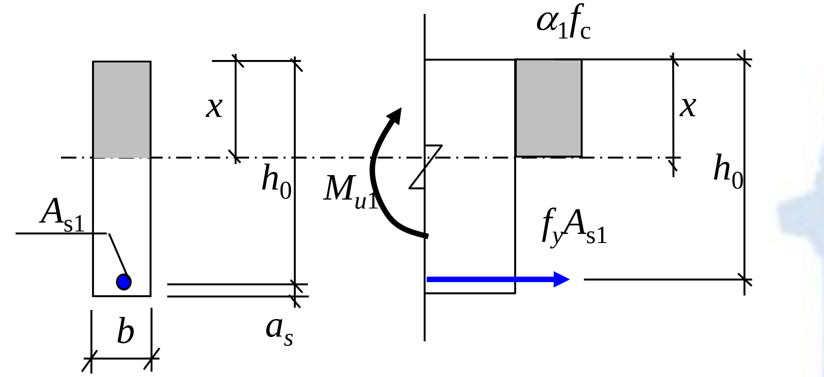
II类T形截面

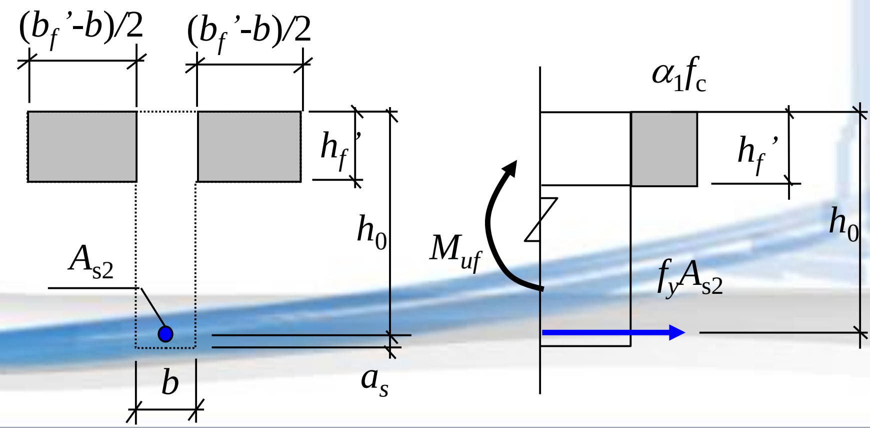
⇩

$$M_{uf}' = \alpha_1 f_c (b_f' - b) h_f' \left(h_0 - \frac{h_f'}{2}\right)$$

⇩

按$b \times h$的单筋矩形截面计算M_{u1}



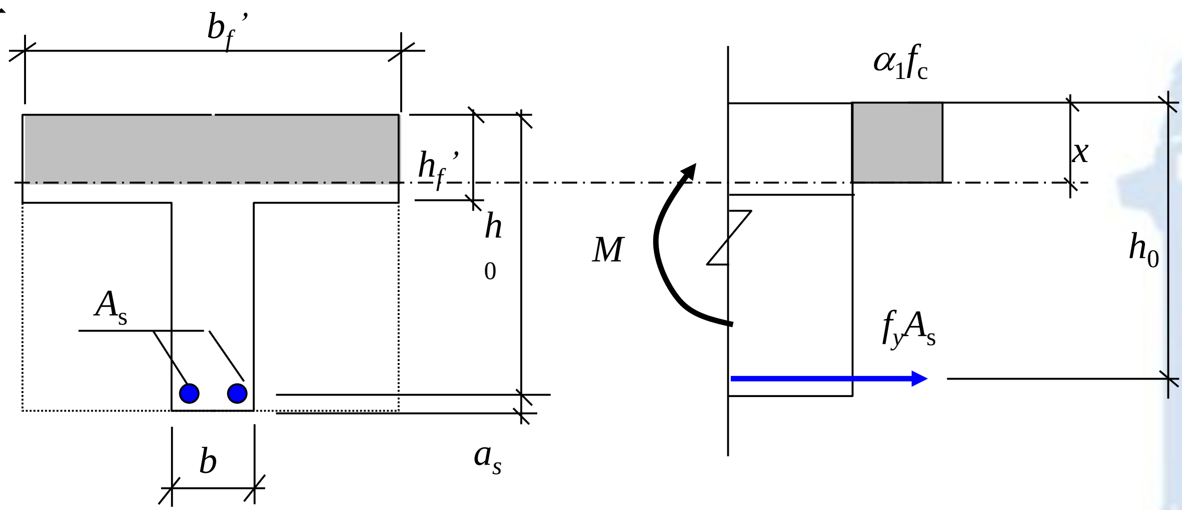


➢T形截面受弯构件

- **简化计算方法**

基于承载力的截面设计

$$M \le \alpha_1 f_c b_f' h_f' (h_0 - \frac{h_f'}{2})$$



I类T形截面

按$b_f' \times h$单筋矩形截面进行设计

$$\rho = \frac{A_s}{bh} \ge \rho_{min}$$

➢T形截面受弯构件

• 简化计算方法

基于承载力的截面设计

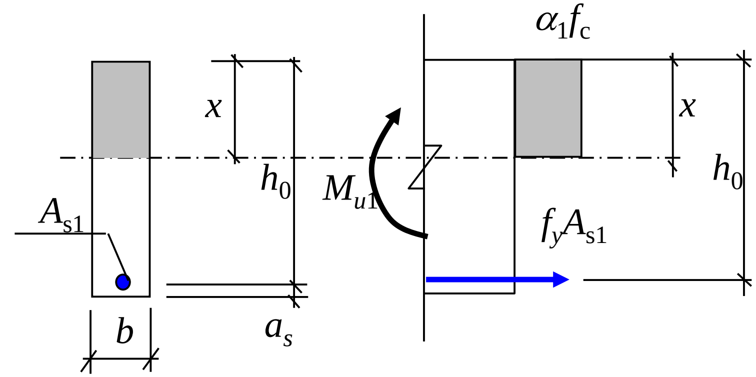
$$M > \alpha_1 f_c b'_f h'_f \left(h_0 - \frac{h'_f}{2}\right)$$

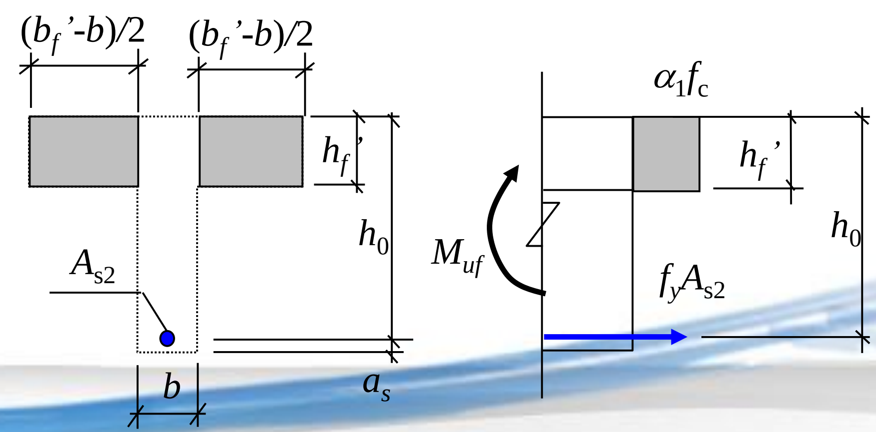
⬇

II类T形截面

⬇

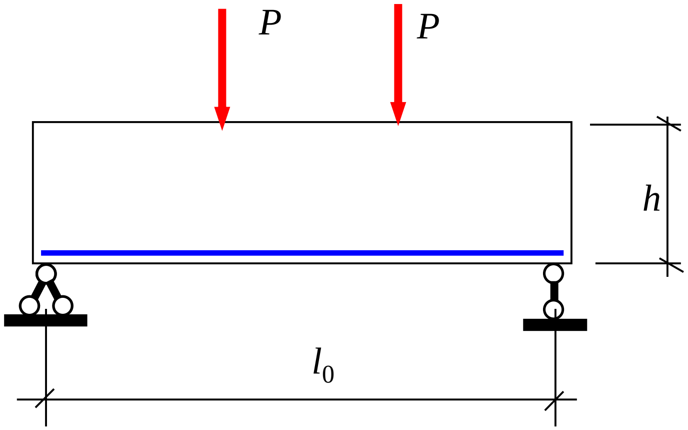
与A_s'已知的$b\times h$双筋矩形截面类似进行设计





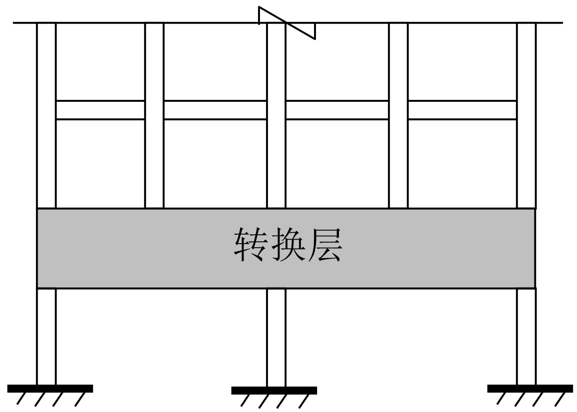
➢深受弯构件

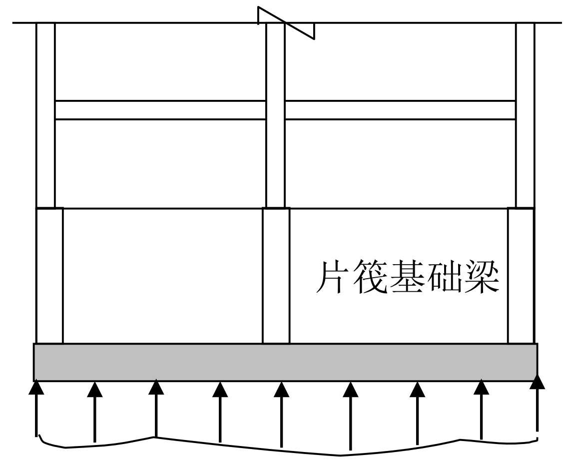
$l_0 / h \leq 5$ ⇨ 深受弯构件

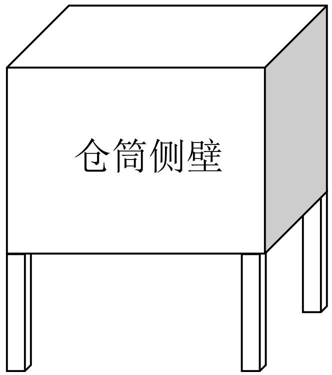


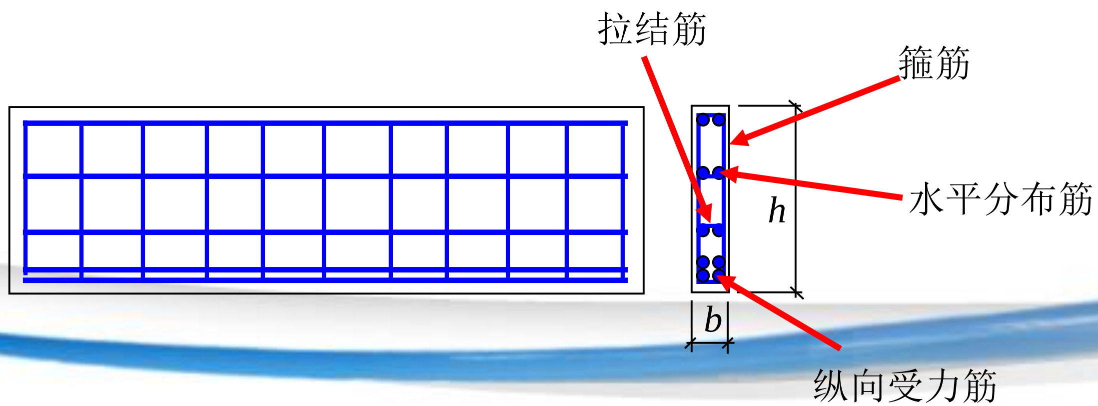
$$\begin{cases} l_0 / h \leq 2.0(\text{简支}),\ l_0 / h \leq 2.5(\text{连续梁}) \rightarrow \text{深梁} \\ 2.0(2.5) < l_0 / h \leq 5 \rightarrow \text{短梁} \end{cases}$$

➢深受弯构件









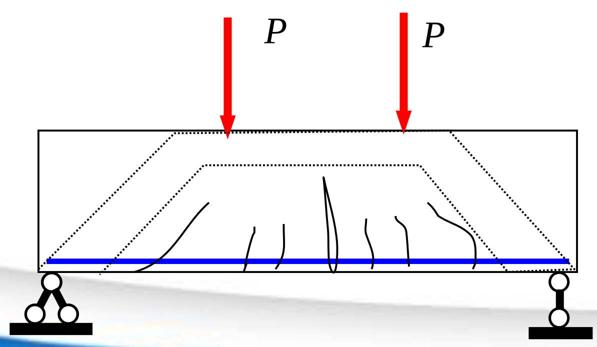
➢深受弯构件

• 深梁的受力性能和破坏形态

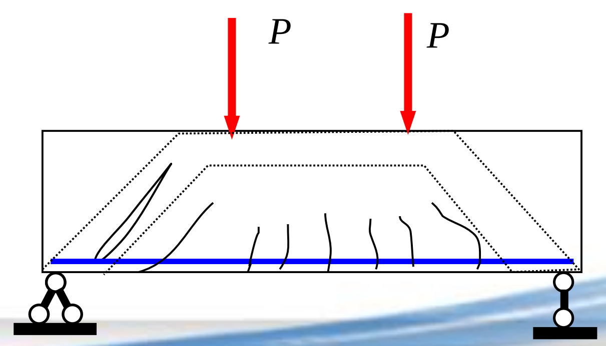
平截面假定不再适用 ➡ 梁的弯曲理论不适用

受力机理 ➡ 拱机理

破坏形态 ➡ 弯曲破坏和剪切破坏(不是此处讨论的内容)



正截面弯曲破坏



斜截面剪切破坏

➢深受弯构件

- 深梁的受力性能和破坏形态

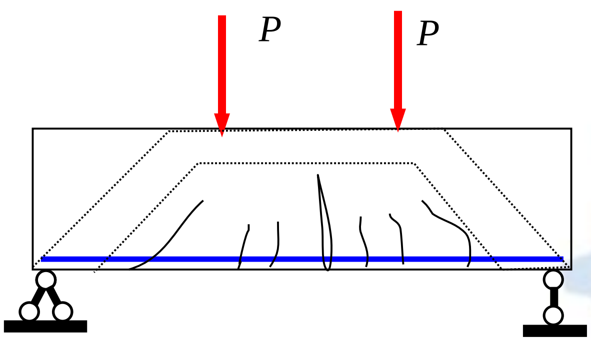
原因：纵筋很少

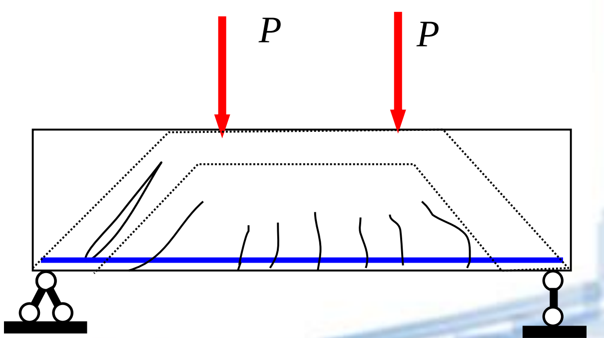
$\rho<\rho_{bm}$时 ➡ 弯曲破坏

以多排受拉钢筋屈服弯矩M_y作为其极限承载力M_u

$\rho>\rho_{bm}$时 ➡ 剪切破坏(此处略)

$\rho=\rho_{bm}$时 ➡ 弯剪界限破坏

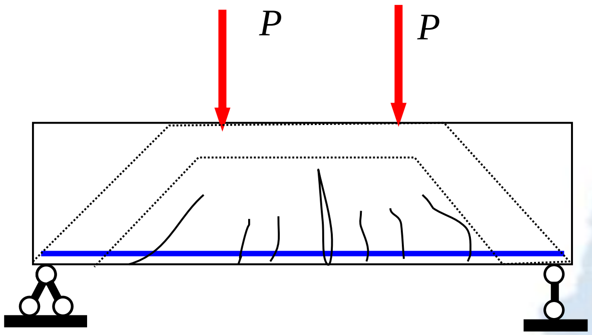




➢深受弯构件

- 深梁的受力性能和破坏形态

由统计回归得出:



简支梁 ➡ $$\rho_{\mathrm{bm}} = 0.19\lambda \frac{f_{\mathrm{c}}}{f_{\mathrm{y}}}$$

计算剪跨比：集中荷载：$\lambda = a/h$

均布荷载：$\lambda = a/h$（$a = l_0/4$）

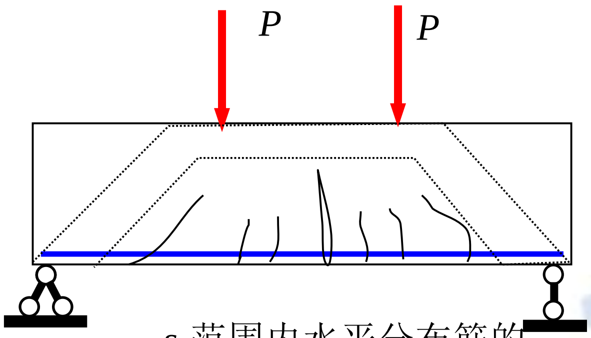
约束梁
连续梁 ➡ $$\rho_{\mathrm{bm}} = \frac{0.19\lambda}{1+1.48\psi} \frac{f_{\mathrm{c}}}{f_{\mathrm{y}}}$$

支座弯矩与跨中最大弯矩的比值绝对值的最大值

➢深受弯构件

• 深梁的受弯承载力

深梁发生弯曲破坏时，截面下部*h*/3范围内的多排钢筋均屈服。由统计回归得出：



$$M_y=(f_yA_s+0.33\rho_h f_{yh}bh)\gamma_s h_0$$

s_v范围内水平分布筋的全部截面积

水平分布筋的配筋率 $\rho_h=\dfrac{A_{sh}}{bs_v}$

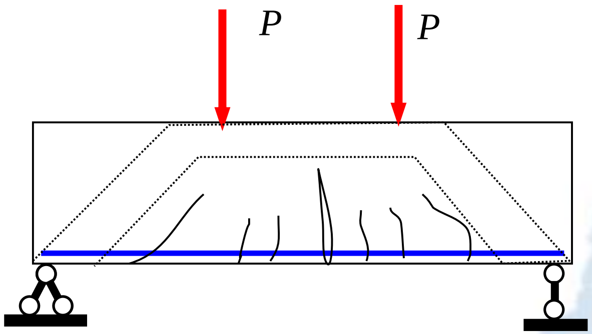
水平分布筋的竖向间距

$$\gamma_s=1-(1-0.1\frac{l_0}{h})(\rho_s+0.5\rho_h\frac{f_{yh}}{f_y})\frac{f_y}{f_c}$$

折算内力臂

➢深受弯构件

• 深梁的受弯承载力



"钢筋混凝土深梁设计规程"（CECS39：92）

简化公式

$$M_u = f_y A_s z$$

深梁的内力臂，取受拉钢筋合力作用点和混凝土受压合力作用点间的距离

简支梁和连续梁的跨中截面 $\quad z = 0.1(l_0 + 5.5h) \qquad (l_0 < h$时，$z = 0.65 l_0)$

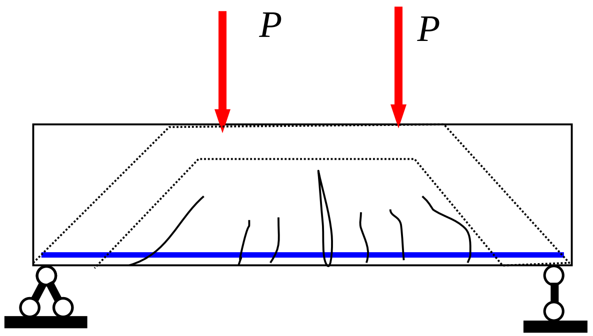
连续梁的支座截面 $\quad z = 0.1(l_0 + 5h) \qquad (l_0 < h$时，$z = 0.6 l_0)$

计算跨度 $\quad l_0 = \min(l_c, 1.15 l_n)$

➢深受弯构件

• 短梁的受弯承载力

和一般梁比较接近，平截面假定适用



破坏类型：少筋、适筋、超筋

适筋梁的受弯承载力

$$M_y = f_y A_s h_0 (0.9 - 0.33\xi)$$

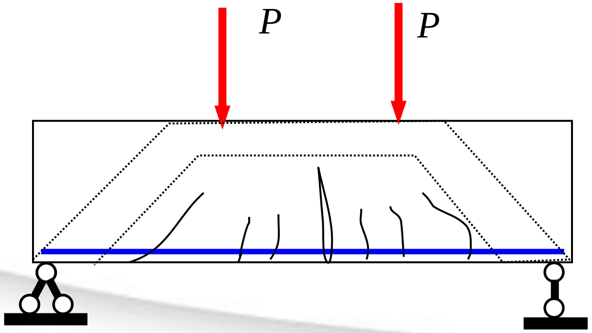
➢深受弯构件

- 规范公式

$$\begin{cases} \alpha_1 f_c bx = f_y A_s \\ M_u = f_y A_s \alpha_d (h_0 - 0.5x) \end{cases}$$

截面有效高度 $h_0 = h - a_s$ $\begin{cases} \dfrac{l_0}{h} < 2\text{时}, \begin{cases} \text{跨中}a_s = 0.1h \\ \text{支座}a_s = 0.2h \end{cases} \\ \dfrac{l_0}{h} > 2\text{时，}a_s\text{受拉纵向钢筋合力作用点至受拉区边缘的距离} \end{cases}$

深受弯构件的内力臂修正系数 $\alpha_d = 0.8 + 0.04\dfrac{l_0}{h}$

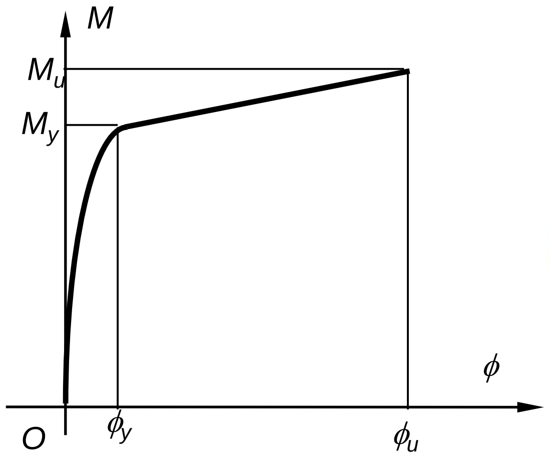


➢受弯构件延性的基本概念

延性 ➡ 反映截面、构件、结构钢筋屈服以后的变形能力

以截面为例：用延性系数表示截面的延性

$$\mu = \varphi_u / \varphi_y$$

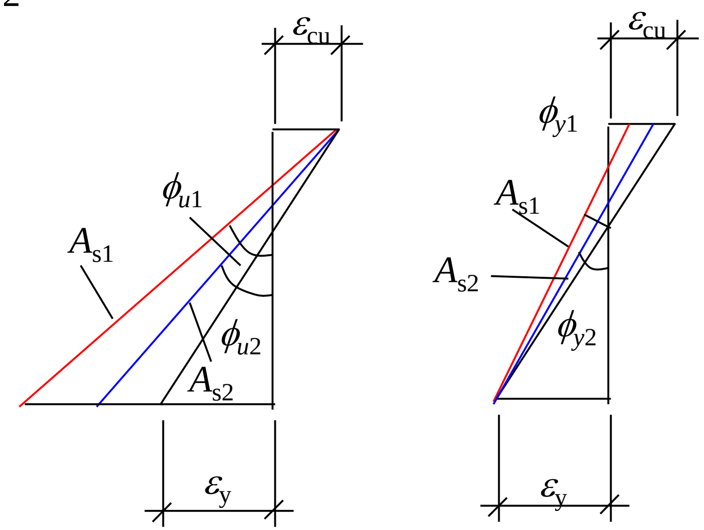


➢受弯构件延性的基本概念

$A_{s1} < A_{s2} \Rightarrow \varphi_{u1} > \varphi_{u2} \quad \varphi_{y1} < \varphi_{y2}$

$$\Downarrow$$

$$\mu_1 = \varphi_{u1} / \varphi_{y1} > \mu_2 = \varphi_{u2} / \varphi_{y2}$$



结构的延性 $\xrightarrow{\text{取决于}}$ 构件的延性 $\xrightarrow{\text{取决于}}$ 截面的延性 $\xrightarrow{\text{取决于}}$ 配筋量