有限单元法研究生核心课程

第十讲 平板壳单元（上）

任晓丹

rxdtj@tongji.edu.cn

同济大学土木工程学院

土木工程学院

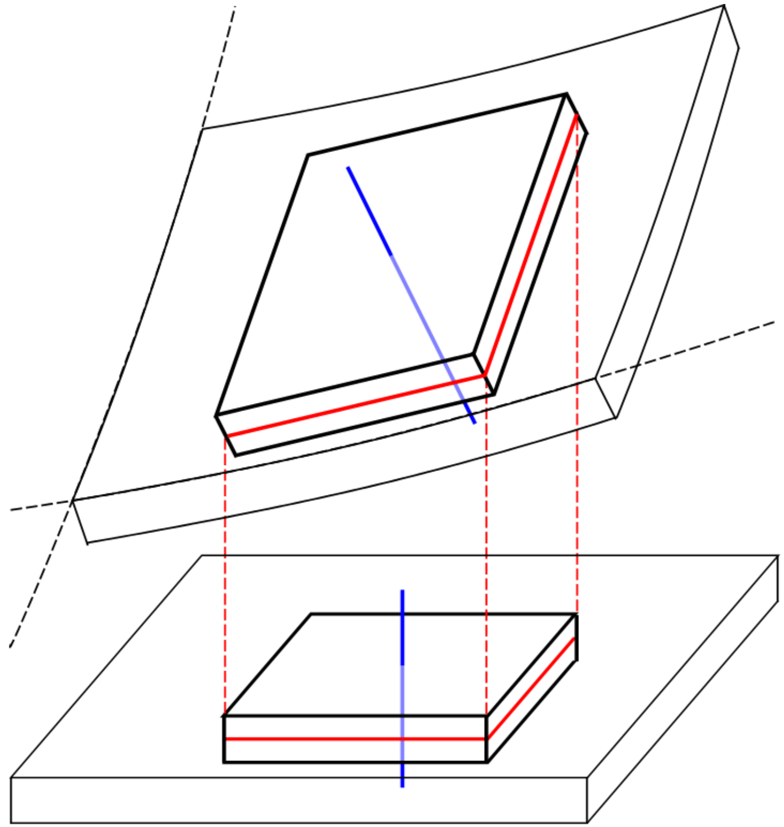
COLLEGE OF CIVIL ENGINEERING

薄板理论

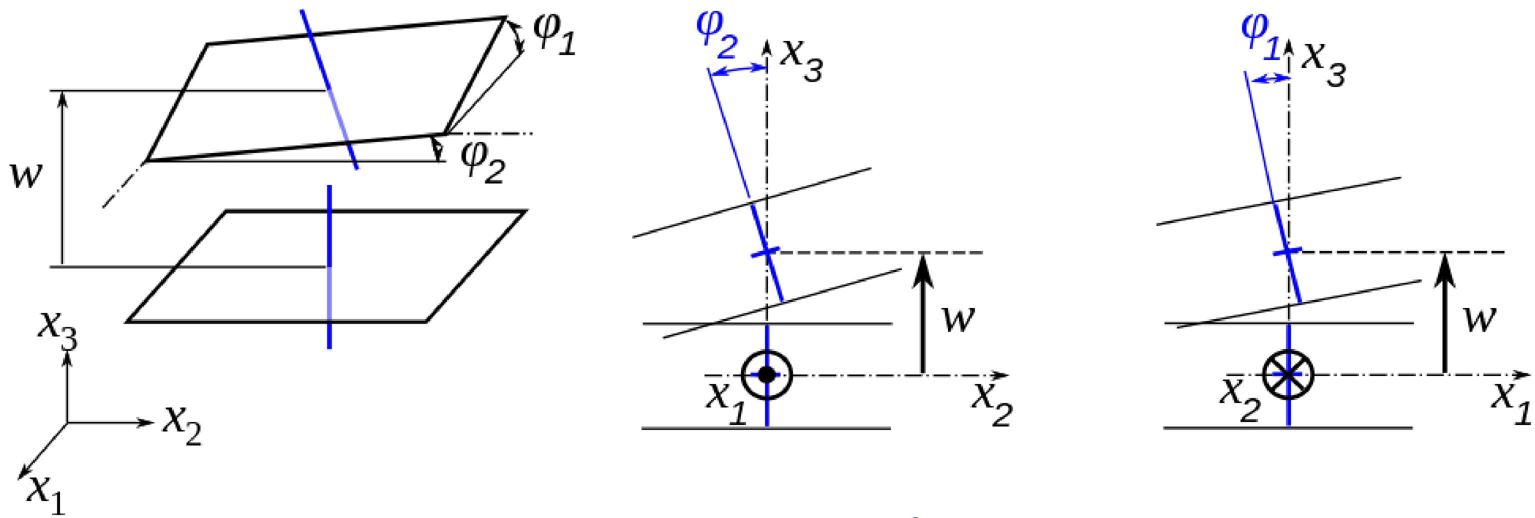
Kirchhoff–Love theory of plates

KIRCHHOFF假定

- 垂直与板中性面的直线变形后仍保持直线并垂直与中性面。
- 板的厚度变形前后保持不变。



薄板的位移



$$u_3 = w$$

$$u_1 = u_1^0 - x_3\theta_1 = u_1^0 - x_3\frac{\partial w}{\partial x_1} = u_1^0 - x_3 w_{,1}$$

$$u_2 = u_2^0 - x_3\theta_2 = u_2^0 - x_3\frac{\partial w}{\partial x_2} = u_2^0 - x_3 w_{,2}$$

薄板的应变

$$\varepsilon_{11} = \frac{\partial u_1}{\partial x_1} = \frac{\partial u_1^0}{\partial x_1} - x_3 w_{,11} \qquad \varepsilon_{22} = \frac{\partial u_2}{\partial x_2} = \frac{\partial u_2^0}{\partial x_2} - x_3 w_{,22}$$

$$\varepsilon_{12} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right) = \frac{1}{2}\left(\frac{\partial u_1^0}{\partial x_2} + \frac{\partial u_2^0}{\partial x_1}\right) - x_3 w_{,12}$$

$$\varepsilon_{31} = \frac{1}{2}\left(\frac{\partial u_3}{\partial x_1} + \frac{\partial u_1}{\partial x_3}\right) = \frac{1}{2}\left(w_{,1} - w_{,1} - x_3 \frac{\partial w_{,1}}{\partial x_3}\right) = 0$$

$$\varepsilon_{32} = \frac{1}{2}\left(\frac{\partial u_3}{\partial x_2} + \frac{\partial u_2}{\partial x_3}\right) = 0 \quad \varepsilon_{33} = \frac{\partial u_3}{\partial x_3} = w_{,3} = 0$$

薄板的应变

$$\varepsilon_{ij}^{0}=\frac{1}{2}\left(\frac{\partial u_{i}^{0}}{\partial x_{j}}+\frac{\partial u_{j}^{0}}{\partial x_{i}}\right) \qquad i,j=1 \text{ or } 2$$

$$\varepsilon_{ij}=\varepsilon_{ij}^{0}-x_{3}w_{,ij} \qquad i,j=1 \text{ or } 2$$

$$\varepsilon_{ij}=0 \qquad i \text{ or } j=3$$

薄板（中性面）的变形

$$\varepsilon_{ij} = \varepsilon_{ij}^{0} - x_3 w_{,ij} \qquad i, j = 1 \text{ or } 2$$

平面内变形　　　弯曲变形

$$\theta_i = -w_{,i}$$ 转角

$$\kappa_{ij} = -\frac{\partial \theta_i}{\partial x_j} = -w_{,ij}$$

$i=j$ 曲率
$i \neq j$ 扭率

薄板应力和内力

薄板应力：

$\sigma_{ij} \quad i, j = 1 \text{ or } 2$ 与应变分量对应

σ_{13}, σ_{23} 截面剪应力 σ_{33} 板表面压力

薄板内力：

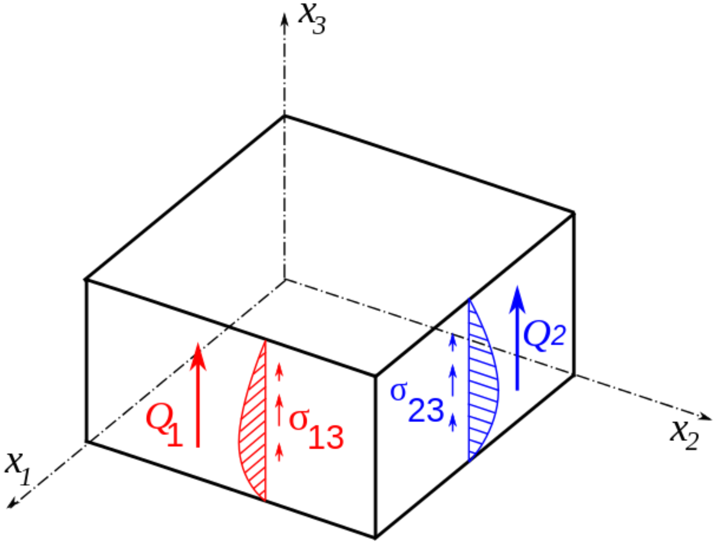
$$N_{ij} = \int_{-h/2}^{h/2} \sigma_{ij} dx_3$$ 薄膜力

$$M_{ij} = \int_{-h/2}^{h/2} \sigma_{ij} x_3 dx_3$$ 弯矩、扭矩

$$Q_{13} = \int_{-h/2}^{h/2} \sigma_{13} dx_3 \quad Q_{23} = \int_{-h/2}^{h/2} \sigma_{23} dx_3$$ 剪力

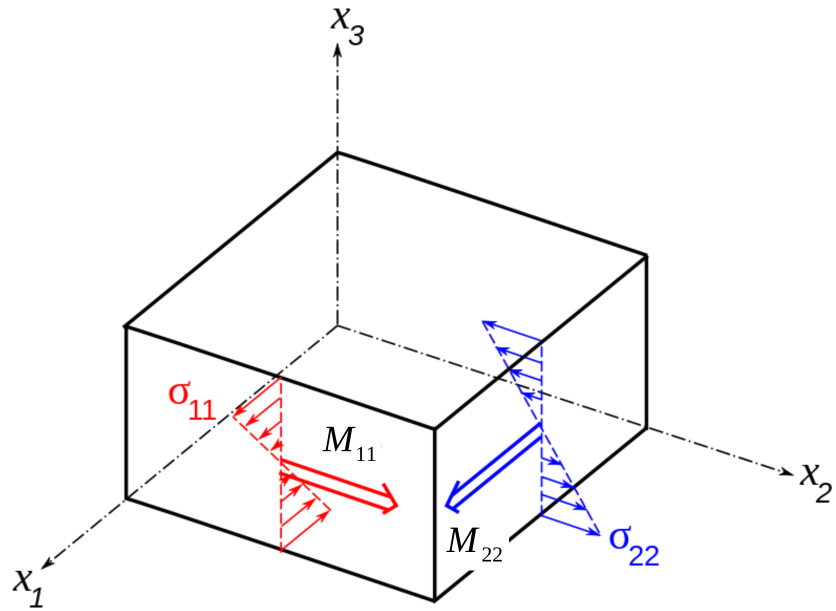
土木工程学院
COLLEGE OF CIVIL ENGINEERING

板的内力



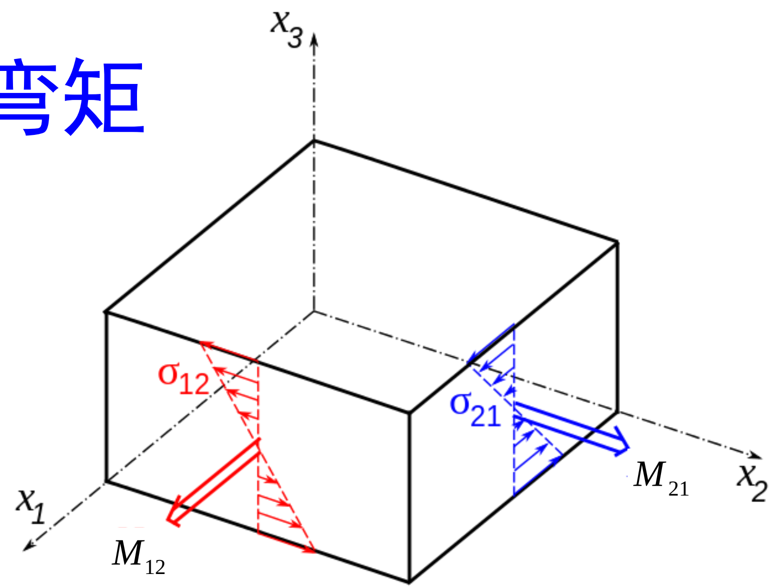
x_3
x_2
x_1
Q_1
σ_{13}
σ_{23}
Q_2

剪力



x_3
x_2
x_1
σ_{11}
M_{11}
M_{22}
σ_{22}

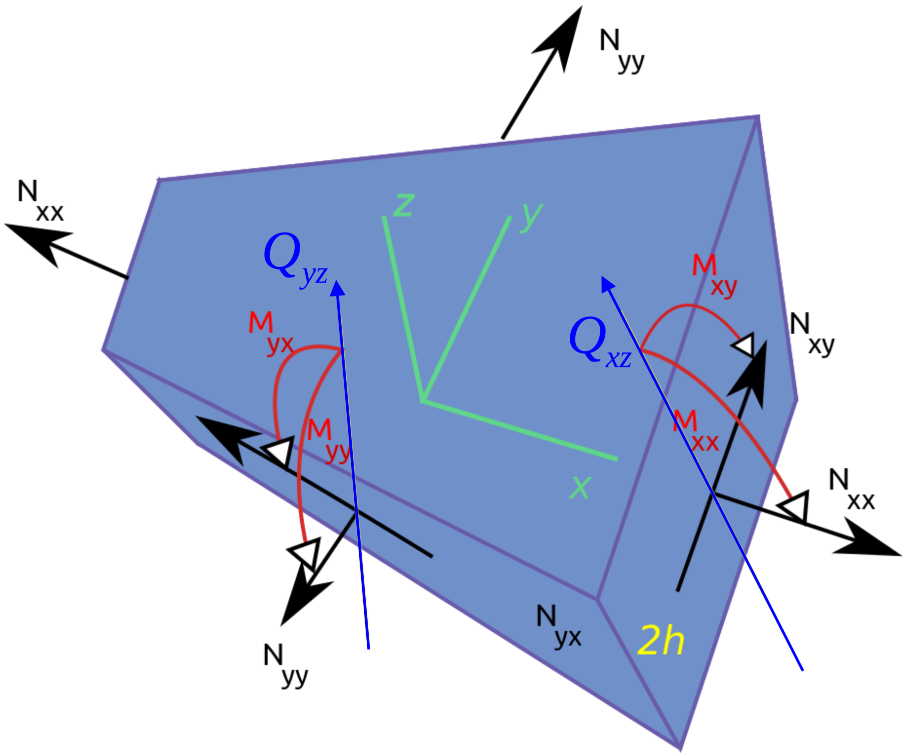
弯矩



x_3
x_2
x_1
σ_{12}
M_{12}
σ_{21}
M_{21}

扭矩

薄板平衡方程



薄膜力平衡

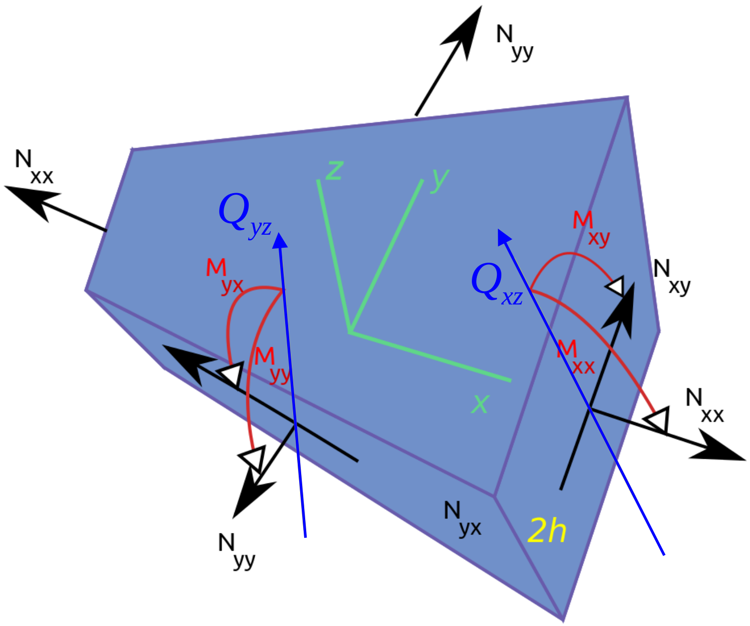
$$N_{ij,j} = \int_{-h/2}^{h/2} \sigma_{ij,j} dx_3 = 0$$

$$i, j = 1 \text{ or } 2$$

$$\frac{\partial N_{11}}{\partial x_1} + \frac{\partial N_{21}}{\partial x_2} = 0$$

$$\frac{\partial N_{12}}{\partial x_1} + \frac{\partial N_{22}}{\partial x_2} = 0$$

薄板平衡方程



弯曲内力平衡

$$\frac{\partial Q_{13}}{\partial x_1}+\frac{\partial Q_{23}}{\partial x_2}=q$$

$$\boxed{\begin{aligned}\frac{\partial M_{11}}{\partial x_1}+\frac{\partial M_{12}}{\partial x_2}&=Q_{13}\\ \frac{\partial M_{21}}{\partial x_1}+\frac{\partial M_{22}}{\partial x_2}&=Q_{23}\end{aligned}}$$

$$\frac{\partial^2 M_{11}}{\partial {x_1}^2}+2\frac{\partial^2 M_{12}}{\partial x_1\partial x_2}+\frac{\partial^2 M_{22}}{\partial {x_2}^2}=q$$

$$M_{ij,ij}=q \qquad i,j=1 \text{ or } 2$$

应力应变关系

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{bmatrix}=\begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ 2\varepsilon_{12}\end{bmatrix}$$

γ_{12}

$$\begin{bmatrix}N_{11}\\ N_{22}\\ N_{12}\end{bmatrix}=\int_{-h/2}^{h/2}\begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ 2\varepsilon_{12}\end{bmatrix}dx_3=h\begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}\begin{bmatrix}\varepsilon_{11}^0\\ \varepsilon_{22}^0\\ 2\varepsilon_{12}^0\end{bmatrix}$$

$$\begin{bmatrix}M_{11}\\ M_{22}\\ M_{12}\end{bmatrix}=\int_{-h/2}^{h/2}\begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ 2\varepsilon_{12}\end{bmatrix}x_3dx_3=-\int_{-h/2}^{h/2}x_3^2dx_3\begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}\begin{bmatrix}w_{,11}\\ w_{,22}\\ 2w_{,12}\end{bmatrix}$$

$$=\frac{h^3}{12}\begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}\begin{bmatrix}\kappa_{11}\\ \kappa_{22}\\ 2\kappa_{12}\end{bmatrix}$$

应力应变关系

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{bmatrix}=\begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ 2\varepsilon_{12}\end{bmatrix}\xRightarrow{\text{各向同性}}\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{bmatrix}=\frac{E}{1-\nu^2}\begin{bmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & (1-\nu)/2\end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ 2\varepsilon_{12}\end{bmatrix}$$

$$\begin{bmatrix}N_{11}\\ N_{22}\\ N_{12}\end{bmatrix}=\frac{Eh}{1-\nu^2}\begin{bmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & (1-\nu)/2\end{bmatrix}\begin{bmatrix}\varepsilon_{11}^0\\ \varepsilon_{22}^0\\ 2\varepsilon_{12}^0\end{bmatrix}\xRightarrow{\text{中性面}}\begin{bmatrix}\sigma_{11}^0\\ \sigma_{22}^0\\ \sigma_{12}^0\end{bmatrix}=\frac{E}{1-\nu^2}\begin{bmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & (1-\nu)/2\end{bmatrix}\begin{bmatrix}\varepsilon_{11}^0\\ \varepsilon_{22}^0\\ 2\varepsilon_{12}^0\end{bmatrix}$$

$$\begin{bmatrix}M_{11}\\ M_{22}\\ M_{12}\end{bmatrix}=-\frac{Eh^3}{12\left(1-\nu^2\right)}\begin{bmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & (1-\nu)/2\end{bmatrix}\begin{bmatrix}w_{,11}\\ w_{,22}\\ 2w_{,12}\end{bmatrix}=D_0\begin{bmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & (1-\nu)/2\end{bmatrix}\begin{bmatrix}\kappa_{11}\\ \kappa_{22}\\ 2\kappa_{12}\end{bmatrix}$$

代入平衡方程

薄膜力平衡 $$N_{ij,j}=0 \Rightarrow \sigma^{0}_{ij,j}=0$$

弯曲内力平衡 $$\frac{\partial^2 M_{11}}{\partial x_1{}^2}+2\frac{\partial^2 M_{12}}{\partial x_1 \partial x_2}+\frac{\partial^2 M_{22}}{\partial x_2{}^2}=q$$

$$\frac{\partial^2 M_{11}}{\partial x_1{}^2}=-D_0\left(w_{,1111}+\nu w_{,2211}\right)$$

$$\frac{\partial^2 M_{22}}{\partial x_2{}^2}=-D_0\left(w_{,2222}+\nu w_{,1122}\right)$$

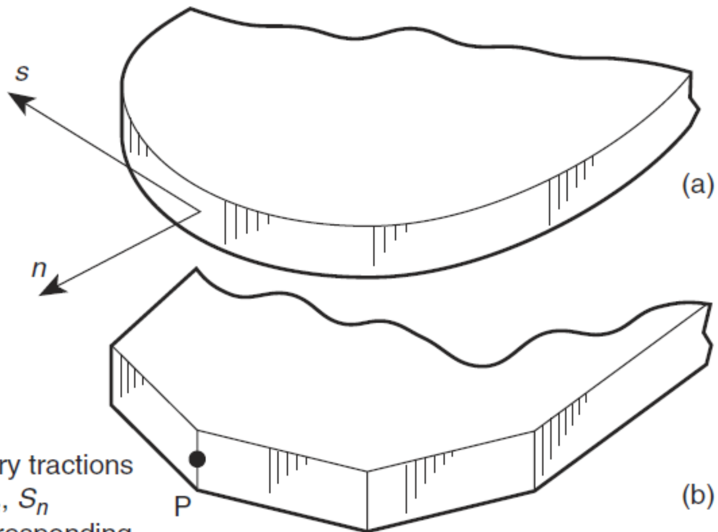
$$\frac{\partial^2 M_{12}}{\partial x_1 \partial x_2}=-D_0(1-\nu)w_{,1212}$$

$$\Rightarrow$$

$$\frac{\partial^2 w_{,11}}{\partial x_1{}^2}+2\frac{\partial^2 w_{,12}}{\partial x_1 \partial x_2}+\frac{\partial^2 w_{,22}}{\partial x_2{}^2}=-\frac{q}{D_0}$$

$$\nabla^2\nabla^2 w=-\frac{q}{D_0}$$

边界条件



Boundary tractions M_n, M_{ns}, S_n and corresponding displacements θ_n, θ_s, w

$$\text{on} \quad S$$

$$\mathbf{n}, \mathbf{s} \rightarrow (n, s)$$

(1) $w\big|_S = \bar{w}$

(2) $\theta_n\big|_S = \bar{\theta} \quad \Rightarrow \quad \left.\dfrac{\partial w}{\partial n}\right|_S = \bar{\theta}$

(3) $V\big|_S = \bar{V}$

(4) $M_n\big|_S = \bar{M}$

(1),(2) (1),(4) (3),(4)

薄板单元

Thin plate element

弱形式

$$\int_{\Omega}\delta w\left(\nabla^2\nabla^2 w-\frac{q}{D_0}\right)\mathrm{d}\Omega=0 \quad \delta w=0 \quad \frac{\partial\delta w}{\partial n}=0$$

相应位移边界

势能泛函

$$\Pi=\int_{\Omega}\left(\frac{1}{2}\boldsymbol{\kappa}^{\mathrm{T}}\mathbf{D}\boldsymbol{\kappa}\right)\mathrm{d}\Omega-\int_{\Omega}qw\mathrm{d}\Omega-\int_{\Gamma_V}\bar{V}w dS+\int_{\Gamma_M}\bar{M}\frac{\partial w}{\partial n}dS$$

$$\delta\Pi=\int_{\Omega}\delta\boldsymbol{\kappa}^{\mathrm{T}}\mathbf{D}\boldsymbol{\kappa}\mathrm{d}\Omega-\int_{\Omega}q\delta w\mathrm{d}\Omega-\int_{\Gamma_V}\bar{V}\delta w dS+\int_{\Gamma_M}\bar{M}\frac{\partial\delta w}{\partial n}dS=0$$

$$w=\mathbf{N}\mathbf{a}^e$$

$$\mathbf{K}\mathbf{a}^e=\mathbf{P}$$

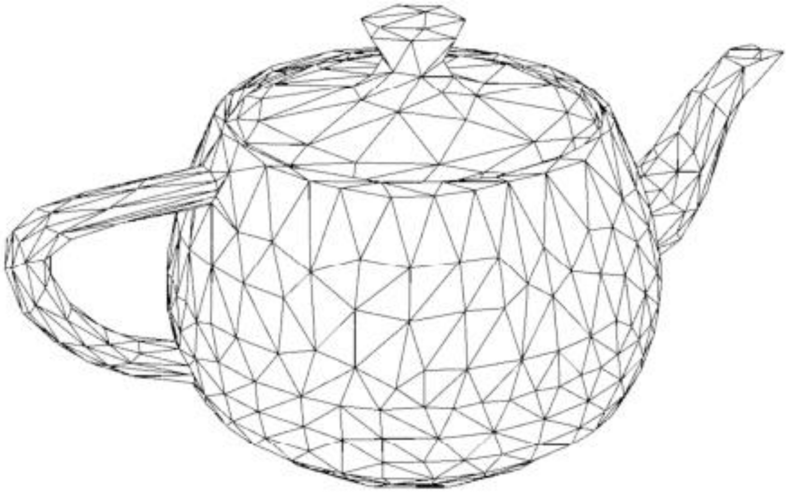
土木工程学院

COLLEGE OF CIVIL ENGINEERING

单元构造

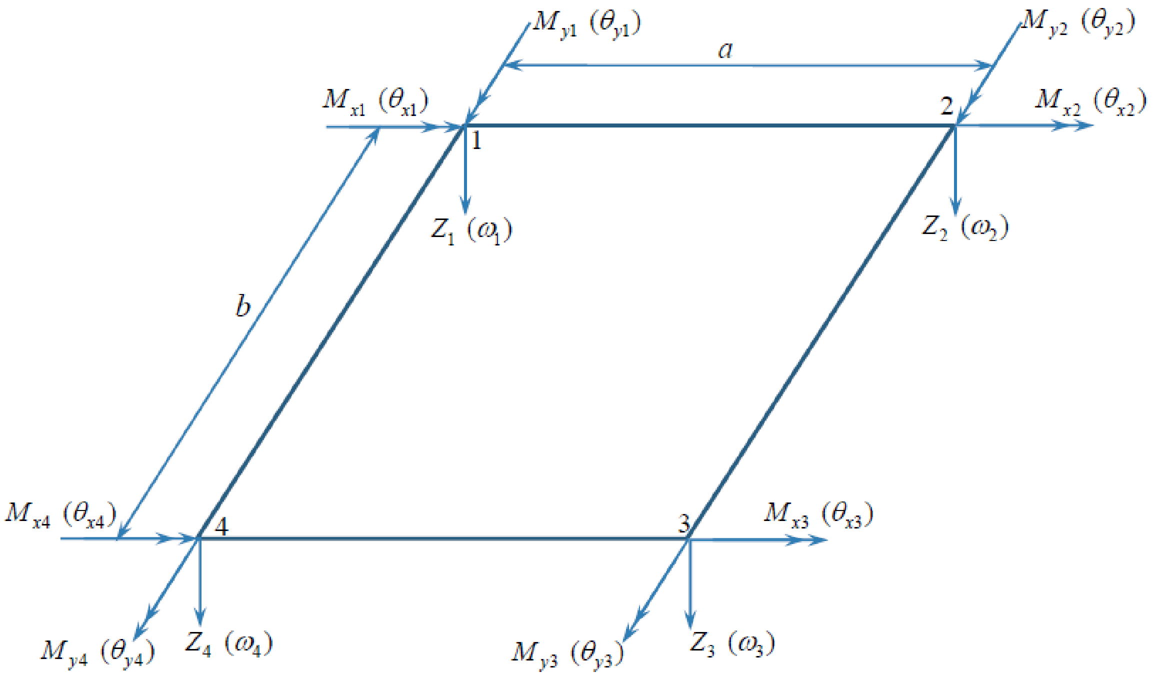
$$w = \mathbf{N}\mathbf{a}^e \quad \boldsymbol{\kappa} = -\begin{bmatrix} w_{,11} & w_{,22} & 2w_{,12} \end{bmatrix}^{\mathrm{T}} = \mathbf{L}w$$

w的二阶导数，
需要C1连续性！



有限元网格很难实现！

四节点（矩形）薄板单元



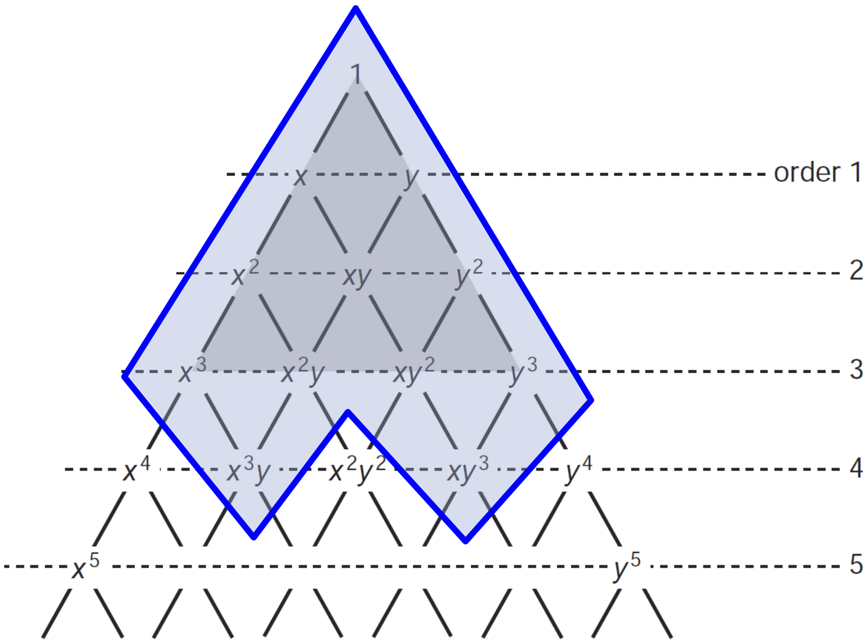
$$\mathbf{a}_i = \begin{bmatrix} w_i \\ \theta_{xi} \\ \theta_{yi} \end{bmatrix} = \begin{bmatrix} w_i \\ \partial w/\partial x|_i \\ -\partial w/\partial y|_i \end{bmatrix}$$

$$i = 1, 2, 3, 4$$

$$\mathbf{a}^e = \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \mathbf{a}_3 \\ \mathbf{a}_4 \end{bmatrix}$$

四节点薄板单元，3×4=12自由度

四节点（矩形）薄板单元



$$
\begin{aligned}
w = {} & \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 x^2 + \alpha_5 xy + \alpha_6 y^2 \\
& + \alpha_7 x^3 + \alpha_8 x^2 y + \alpha_9 xy^2 + \alpha_{10} y^3 \\
& + \alpha_{11} x^3 y + \alpha_{12} xy^3 \\
= {} & \mathbf{P}\boldsymbol{\alpha}
\end{aligned}
$$

$$\mathbf{P}(x_i, y_i)\boldsymbol{\alpha} = w_i$$

$$\left.\frac{\partial \mathbf{P}}{\partial x}\right|_{(x_i, y_i)} \boldsymbol{\alpha} = \theta_{xi}$$

$$-\left.\frac{\partial \mathbf{P}}{\partial y}\right|_{(x_i, y_i)} \boldsymbol{\alpha} = \theta_{yi}$$

联立12个方程
解得插值系数

四节点薄板单元，3×4=12自由度

四节点（矩形）薄板单元

$$\mathbf{N}_i^{\mathrm{T}} = \frac{1}{8}\begin{bmatrix} (\xi\xi_i+1)(\eta\eta_i+1)(2+\xi\xi_i+\eta\eta-\xi^2-\eta^2) \\ b\eta_i(\xi\xi_i+1)(\eta\eta_i+1)^2(\eta\eta_i-1) \\ -a\xi_i(\xi\xi_i+1)^2(\xi\xi_i-1)(\eta\eta_i+1) \end{bmatrix}$$

$$\boldsymbol{\kappa} = \mathbf{L}w = \mathbf{L}\mathbf{N}\mathbf{a}^e = \mathbf{B}\mathbf{a}^e \qquad \mathbf{M} = \mathbf{D}\boldsymbol{\kappa} = \mathbf{D}\mathbf{B}\mathbf{a}^e$$

$$\mathbf{K}^e = \int_{\Omega^e} \mathbf{B}^{\mathrm{T}}\mathbf{D}\mathbf{B}\,\mathrm{d}\Omega \qquad \mathbf{P}^e = \int_{\Omega^e} \mathbf{N}^{\mathrm{T}} q\,\mathrm{d}\Omega - \int_{\Gamma_V^e} \mathbf{N}^{\mathrm{T}}\bar{V}dS + \int_{\Gamma_M^e} \left(\frac{\partial \mathbf{N}}{\partial n}\right)^{\mathrm{T}} \bar{M}dS$$

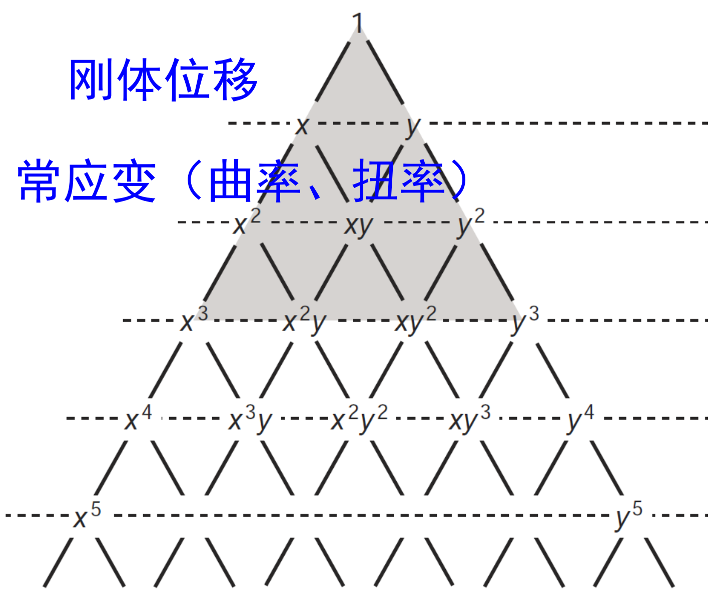
$$\mathbf{K} = \mathop{\mathsf{A}}_{e=1}^{n_{el}} \mathbf{K}^e, \mathbf{P} = \mathop{\mathsf{A}}_{e=1}^{n_{el}} \mathbf{P}^e \qquad \mathbf{K}\mathbf{a} = \mathbf{P}$$

四节点（矩形）薄板单元

$$w=\alpha_1+\alpha_2x+\alpha_3y+\alpha_4x^2+\alpha_5xy+\alpha_6y^2+\alpha_7x^3$$
$$+\alpha_8x^2y+\alpha_9xy^2+\alpha_{10}y^3+\alpha_{11}x^3y+\alpha_{12}xy^3$$

刚体位移

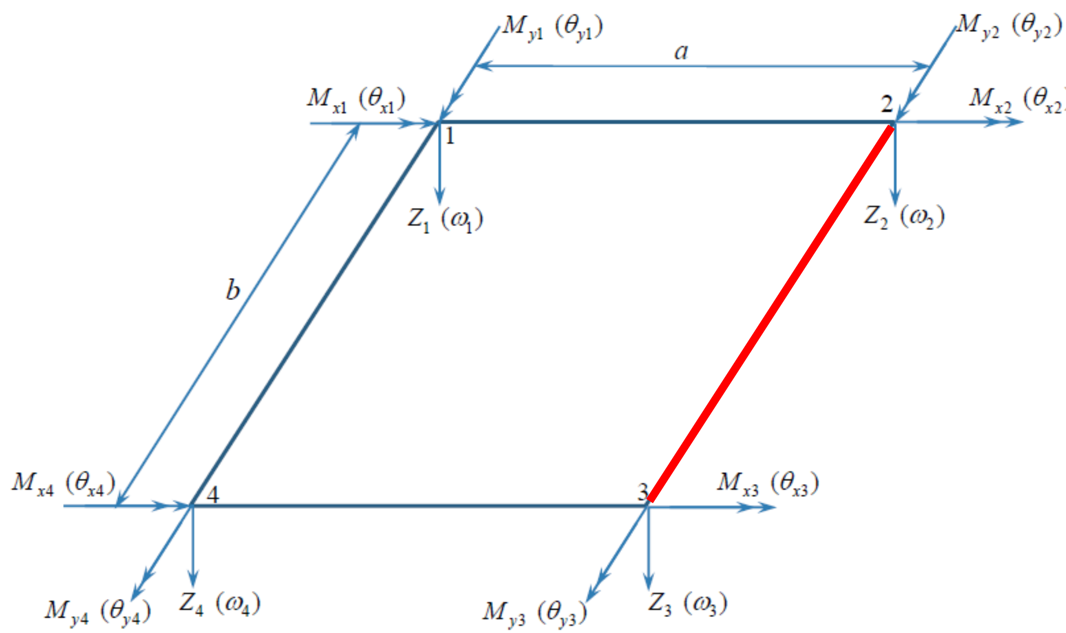
常应变（曲率、扭率）



满足完备性要求

不能推广到（等参）四边形板元

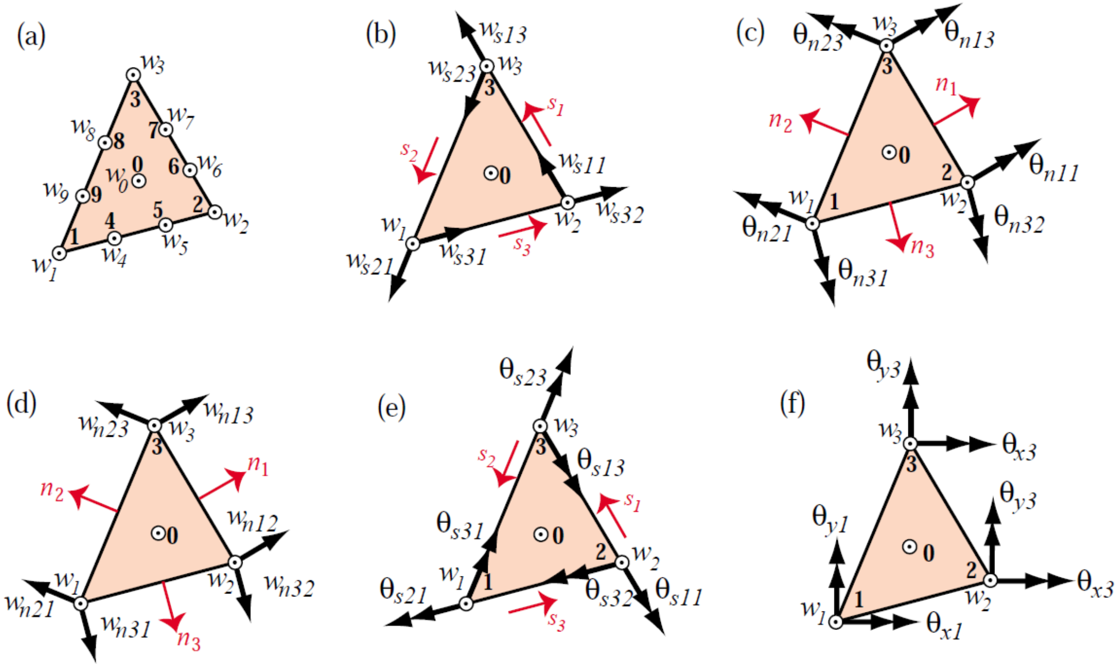
平行四边形板元尚可

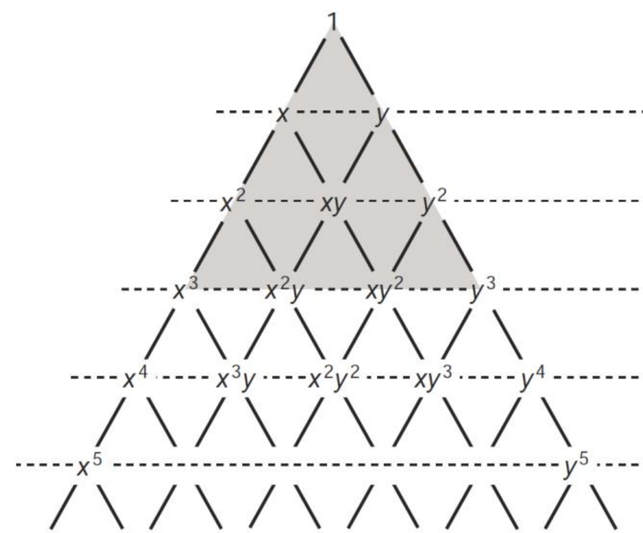


满足挠度协调性要求

不满足（转角）协调性要求

三节点薄板单元





采用待定系数法很难取舍！

三节点薄板单元自由度（9个物理自由度）

三节点薄板单元

面积坐标及其多项式：

一次项： L_1, L_2, L_3　　一次完全多项式： $\alpha_1 L_1 + \alpha_2 L_2 + \alpha_3 L_3$

二次项： $L_1^2, L_2^2, L_3^2, L_1L_2, L_2L_3, L_3L_1$

二次完全多项式为一、二次项中任意6项线性组合，最常用形式：

$$\alpha_1 L_1 + \alpha_2 L_2 + \alpha_3 L_3 + \alpha_4 L_1 L_2 + \alpha_5 L_2 L_3 + \alpha_6 L_3 L_1$$

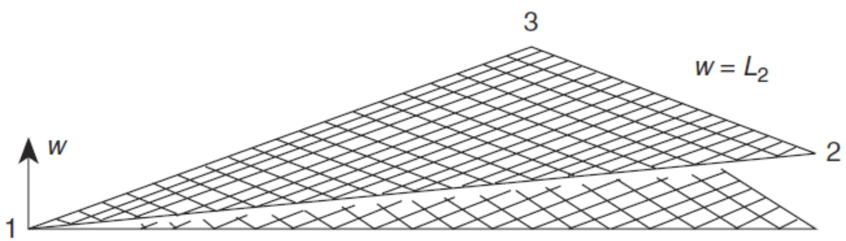
三次项： $L_1^3, L_2^3, L_3^3, L_1^2 L_2, L_2^2 L_3, L_3^2 L_1, L_1 L_2^2, L_2 L_3^2, L_3 L_1^2, L_1 L_2 L_3$

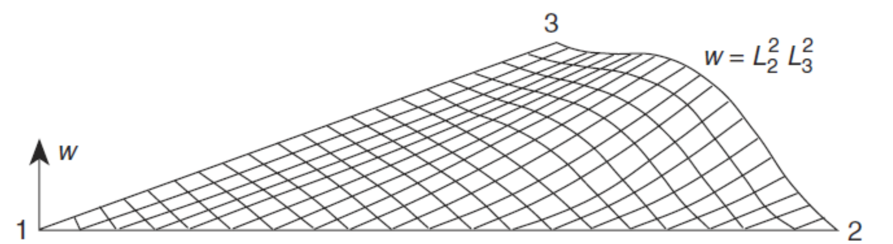
三次完全多项式为一、二、三次项中任意10项线性组合，最常用形式：

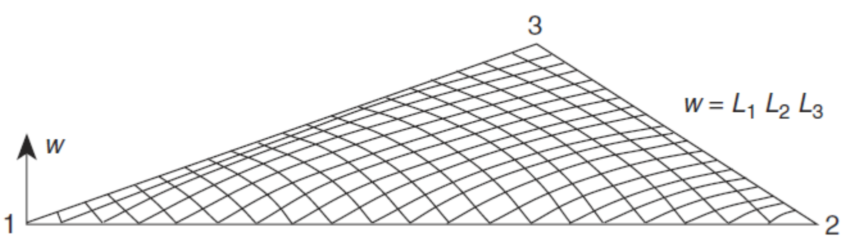
$$\alpha_1 L_1 + \alpha_2 L_2 + \alpha_3 L_3 + \alpha_4 L_1 L_2 + \alpha_5 L_2 L_3 + \alpha_6 L_3 L_1 + \alpha_7 L_1^2 L_2 + \alpha_8 L_2^2 L_3 + \alpha_9 L_3^2 L_1 + \alpha_{10} L_1 L_2 L_3$$

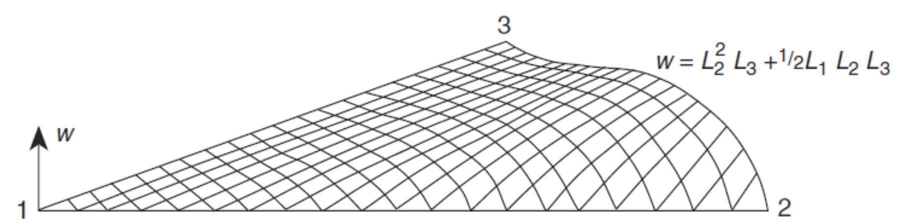
三节点薄板单元

面积坐标插值多项式性质：









Bubble function，与节点位移/转角无关，
可灵活使用使单元满足某些给定性质。

三节点薄板单元

$w = \mathbf{P}\boldsymbol{\alpha}$　　$\mathbf{P}, \boldsymbol{\alpha}$ 各9项

$$\mathbf{P}(x_i, y_i)\boldsymbol{\alpha} = w_i, \left.\frac{\partial \mathbf{P}}{\partial x}\right|_{(x_i, y_i)} \boldsymbol{\alpha} = \theta_{xi}, -\left.\frac{\partial \mathbf{P}}{\partial y}\right|_{(x_i, y_i)} \boldsymbol{\alpha} = \theta_{yi}$$

联立9个方程
解得插值系数

其中面积坐标的偏导数：

$$\begin{bmatrix} \dfrac{\partial}{\partial x} \\ \dfrac{\partial}{\partial y} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial L_1}{\partial x} & \dfrac{\partial L_2}{\partial x} & \dfrac{\partial L_3}{\partial x} \\ \dfrac{\partial L_1}{\partial y} & \dfrac{\partial L_2}{\partial y} & \dfrac{\partial L_3}{\partial y} \end{bmatrix} \begin{bmatrix} \dfrac{\partial}{\partial L_1} \\ \dfrac{\partial}{\partial L_2} \\ \dfrac{\partial}{\partial L_3} \end{bmatrix} = \frac{1}{2A} \begin{bmatrix} b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} \dfrac{\partial}{\partial L_1} \\ \dfrac{\partial}{\partial L_2} \\ \dfrac{\partial}{\partial L_3} \end{bmatrix}$$

三节点薄板单元

$$w=\alpha_1L_1+\alpha_2L_2+\alpha_3L_3+\alpha_4L_1L_2+\alpha_5L_2L_3+\alpha_6L_3L_1+\alpha_7L_1^2L_2+\alpha_8L_2^2L_3+\alpha_9L_3^2L_1$$

$$\Rightarrow\quad \mathbf{N}_i^{\mathrm{T}}=\begin{bmatrix} 3L_i^2-2L_i^3 \\ L_i^2(b_jL_k-b_kL_j) \\ L_i^2(c_jL_k-c_kL_j) \end{bmatrix}$$

不满足常应变要求！

$$\begin{aligned} w=&\alpha_1L_1+\alpha_2L_2+\alpha_3L_3 \\ &+\alpha_4\left(L_1L_2+CL_1L_2L_3\right)+\alpha_5\left(L_2L_3+CL_1L_2L_3\right)+\alpha_6\left(L_3L_1+CL_1L_2L_3\right) \\ &+\alpha_7\left(L_1^2L_2+CL_1L_2L_3\right)+\alpha_8\left(L_2^2L_3+CL_1L_2L_3\right)+\alpha_9\left(L_3^2L_1+CL_1L_2L_3\right) \end{aligned}$$

$$\Rightarrow\quad \mathbf{N}_i^{\mathrm{T}}=\begin{bmatrix} 3L_i^2-2L_i^3 \\ L_i^2(b_jL_k-b_kL_j)+C(b_j-b_k)L_1L_2L_3 \\ L_i^2(c_jL_k-c_kL_j)+C(c_j-c_k)L_1L_2L_3 \end{bmatrix}$$

C=1/2时满足常应变要求！但只对有限的情况精确满足分片试验！

三节点薄板单元

$$\mathbf{P}=\big[L_1 \quad L_2 \quad L_3 \quad L_1L_2 \quad L_2L_3 \quad L_3L_1$$

$$L_1^2L_2+\tfrac{1}{2}L_1L_2L_3\{3(1-\mu_3)L_1-(1+3\mu_3)L_2+(1+3\mu_3)L_3\}$$

$$L_2^2L_3+\tfrac{1}{2}L_1L_2L_3\{3(1-\mu_1)L_1-(1+3\mu_1)L_2+(1+3\mu_1)L_3\}$$

$$L_3^2L+\tfrac{1}{2}L_1L_2L_3\{3(1-\mu_2)L_1-(1+3\mu_2)L_2+(1+3\mu_2)L_3\}\big]$$

$$\mu_i=\frac{l_k^2-l_j^2}{l_i^2}$$

$$\Rightarrow \quad \mathbf{N}_i^{\mathrm{T}}=\begin{bmatrix} P_i-P_{i+3}+P_{k+3}+2(P_{i+6}-P_{k+6}) \\ -b_j(P_{k+6}-P_{k+3})-b_kP_{i+6} \\ -c_j(P_{k+6}-P_{k+3})-c_kP_{i+6} \end{bmatrix}$$

该单元能通过任意形式的分片试验，与三点（减缩）积分配合性能良好。

公共邮箱

地址：fem_tongji@163.com

密码：youxiandanyuanfa

土木工程学院

COLLEGE OF CIVIL ENGINEERING

今天就到这里，

明天的事儿明天再说！

任晓丹

rxdtj@tongji.edu.cn

同济大学土木工程学院